Optimal Portfolio Choice with Annuities and Life Insurance for Retired Couples

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Abstract

This paper derives the optimal asset allocation and consumption policy for a retired couple with uncertain lifetime, possible pre-existing annuity income, and a potential bequest motive. The joint life time utility maximizing household can dynamically adjust liquid retirement assets and purchase term life insurance and single as well as joint annuities at any time and incrementally. Joint annuities are very appealing, since they hedge the couple’s longevity risk without being contingent on the survival of a certain spouse. Our results show that the optimal survivor benefit ratio of joint annuities is closely related to the couple’s degree of jointness in consumption. We find that the demand for life insurance is mostly driven by the aim to protect against the loss of annuity income for the surviving spouse. Especially if pre-existing annuity income is distributed unequally among the partners, the access to life insurance contracts is highly welfare enhancing. In contrast to bonds and risky stocks, term life insurance contracts are not attractive to finance a bequest motive.

Keywords: Household Finance, Portfolio Choice, Life Insurance, Joint Annuities, Retired Couples

JEL Codes: G11, G22, D14, D91

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1. Introduction

Couples in retirement face the challenge to derive consumption and investment policies that will maximize their joint lifetime utility subject to pre-existing pension income and endowments with retirement savings. One of the central risks they face is outliving their retirement assets due to unexpected longevity. While individuals can hedge this risk by purchasing single life annuities, in multi-person households an early death of one partner may result in a substantial drop in annuity income for the surviving spouse. In addition to holding sufficiently large funds in the capital markets, such as bond or stock investments, households can hedge this risk by purchasing life insurance contracts and by adequately combining single and joint life annuities.

Both, life insurance and annuities are financial products which offer the policyholder survival contingent payouts in exchange for an initial nonrefundable premium. A term life insurance contract pays a certain lump sum to the beneficiary, if the policyholder dies within a pre-specified period. A single annuity contract entitles the annuitant to regular payments as long as he is alive, whereas a joint annuity provides regular payments as long as (at least) one of the two beneficiaries is alive. Depending on the agreed upon last survivor payout rule, benefits may decrease by a pre-specified fraction after the decease of one of the beneficiaries.

It is an open question how retired couples should optimally combine these products in their golden years to maximize their joint lifetime utilities. To shed more light on this issue, we develop a dynamic two-person life cycle consumption and portfolio choice model that allows for gradual annuitization with single and joint annuities, repeated purchases of one-period life insurance contracts, as well as investments in risk-free bonds and risky stocks. We find that couples aim at building an annuity portfolio with a symmetric income distribution among the two partners and that an initially asymmetric annuity income distribution leads to demand for term life insurance. Moreover, we are able to endogenously derive the optimal fraction by which joint annuity income is reduced upon the first death. We show that this fraction directly depends on the level of jointness in their consumption.

This paper extends previous research on the impact of longevity risk on life cycle portfolio choice in several directions. Kotlikoff and Spivak (1981) show that the pooling of mortality
risk in a multi-individual household leads to welfare gains comparable to having access to the annuity market. Brown and Poterba (2000) study the welfare gains for couples of having access to joint life annuity products based on the annuity equivalent wealth framework developed by Mitchell et al. (1999). Their findings suggest that these gains can be as high as 70% of retirement wealth, depending on preference parameters and the amount of pre-existing annuity income. Yet, their assumptions are quite restrictive: their model setting only allows for complete annuitization of all accumulated assets at the beginning of the retirement phase, given an exogenously specified survivor benefit factor. Moreover, they do not allow for investments in risky stocks or life insurance contracts, and they only compare welfare gains of an annuitization strategy against a benchmark of risk-free bonds.

Recent studies by Horneff et al. (2008, 2009, 2010) integrate individual annuities into realistically calibrated life cycle portfolio choice models, allowing for risky stock investments and optimal gradual annuitization tactics. They demonstrate that these products are welfare enhancing in that they offer consumers an effective hedge against individual longevity risk as well as the opportunity to trade liquidity for a survival-contingent extra return known as the ‘survival credit’. Yet, they develop their model from the perspective of a single household rather than from the perspective of a couple. Moreover, they do not allow for term life insurance. More recently, Love (2011) extends the Cocco, Gomes, and Maenhout (2005) study by integrating shocks in family size into a dynamic life cycle model. While he allows for term life insurance purchases, single and joint annuities are neglected.

In what follows, we develop the structure of our life cycle optimization model for a retired couple that has access to the various products to manage their consumption and bequest requirements. We then explore and discuss for our base case the role of these products in optimal life cycle profiles. For alternative calibrations we check the robustness of our results. Subsequently, we derive the optimal survivor benefit factor for a joint annuity. A welfare analysis shows the potential gains of having access to life insurance, single and joint annuities. A final section concludes.
2. The life cycle optimization model for a retired couple

2.1 Family dynamics

We model a couple in retirement that faces an uncertain lifespan governed by gender-specific one-year survival probabilities $p^x_t$ and $p^y_t$. Here and in the following, $x$ ($y$) represents the husband (wife) as well as his (her) age and $t=(0,\ldots,T)$ the time after the couple retired (measured in years). We assume that the decease of one spouse does not affect the survival probability of the other spouse.\footnote{We abstract from effects like the ‘broken heart’ syndrome (see e.g. Parkes, Benjamin, and Fitzgerald 1969), as Brown and Poterba (2000) find these to have only minor effect impact on their results.}

At each point in time, the couple may be in either one of four different family states $s_t$: both being alive, wife having deceased (widower), husband having deceased (widow), and both having deceased. Using indicator variables $\mathbb{I}_t^x$ and $\mathbb{I}_t^y$, which are 1 if the wife/husband is alive at time $t$ and zero otherwise, these four states can be represented by

\[
s_t = \begin{cases} 
1 & \mathbb{I}_t^x = 1, \mathbb{I}_t^y = 1 \quad \text{both alive} \\
2 & \mathbb{I}_t^x = 1, \mathbb{I}_t^y = 0 \quad \text{widower} \\
3 & \mathbb{I}_t^x = 0, \mathbb{I}_t^y = 1 \quad \text{widow} \\
4 & \mathbb{I}_t^x = 0, \mathbb{I}_t^y = 0 \quad \text{both deceased} 
\end{cases} \quad (2.1)
\]

The time-dependent transition matrix $\Pi_{i,j,t} = \text{Prob}(s_{t+1} = i \mid s_t = j)$ of this Markov chain is specified by the individual one-year survival probabilities:

\[
\Pi_t = \begin{pmatrix}
    p^x_t \cdot p^y_t & 0 & 0 & 0 \\
    p^x_t \cdot (1 - p^y_t) & p^x_t & 0 & 0 \\
    (1 - p^x_t) \cdot p^y_t & 0 & p^y_t & 0 \\
    (1 - p^x_t)(1 - p^y_t) & 1 - p^x_t & 1 - p^y_t & 1
\end{pmatrix}. \quad (2.2)
\]

At the end of our projection horizon $T$, we set $p^x_T = p^y_T = 0$. Obviously we neglect all not mortality driven family changes such as a divorce or a new life partnership after one of the spouses has died.
2.2 Financial products

The couple can choose among different financial products to manage retirement income and potential bequests: riskless bonds, risky stocks, life insurances and annuities. Bonds have a constant annual real gross rate of return, represented by $R_f$. The annual gross returns of stocks are serially independent and identically lognormally distributed; $R_{t+1}$ represents the return from time $t$ to time $t+1$.

At each time $t$, a one-year term life insurance contract can be purchased for each living spouse $i \in \{x, y\}$. If the insured spouse dies within the period $[t, t+1]$, the insurer will pay the face value $L_t^i$ at time $t+1$. The actuarially fair insurance premium $LP_t^i$ is given by the discounted expected payout

$$LP_t^i = (1 - p_t^i) \cdot \frac{L_t^i}{R_f}. \quad (2.3)$$

Similar to life insurance products, single annuities can be purchased for each spouse separately. We denote by $\bar{A}^i$ a normalized annuity that pays every year the constant amount of one monetary unit as long as the annuitant $i \in \{x, y\}$ is alive. The gender- and age-dependent annuity factor $\bar{a}_t^i$, which is the actuarially fair price of $\bar{A}^i$ at time $t$, is given by

$$\bar{a}_t^i = \sum_{\tau=t+1}^{T} \frac{p_{\tau,t}^i}{(R_f)^{\tau-t}}. \quad (2.4)$$

Here $p_{\tau,t}^i$ denotes the probability that spouse $i$ is alive at time $\tau$ conditional on being alive at time $t$, which is the product $p_{\tau,t}^i = \prod_{s=t}^{\tau-1} p_s^i$ of the one-year survival probabilities.

A normalized joint annuity $\bar{A}^{xy}$ pays one monetary unit as long as at least one of the spouses is alive. The corresponding annuity factor is given by

$$\bar{a}_t^{xy} = \sum_{\tau=t+1}^{T} \frac{p_{\tau,t}^{xy}}{(R_f)^{\tau-t}}. \quad (2.5)$$

Here $p_{\tau,t}^{xy}$ denotes the probability, that at least one of the spouses is alive at time $\tau$, conditional on both being alive at time $t$

$$p_{\tau,t}^{xy} = p_{\tau,t}^x p_{\tau,t}^y + p_{\tau,t}^x (1 - p_{\tau,t}^y) + (1 - p_{\tau,t}^x) p_{\tau,t}^y$$

$$= p_{\tau,t}^x + p_{\tau,t}^y - p_{\tau,t}^x p_{\tau,t}^y. \quad (2.6)$$
2.3 Replication of any joint annuity

Most joint annuities available do not pay a constant amount until the last spouse dies. Typically a survivor benefit ratio \( K \leq 1 \) is chosen, to which the payment level is reduced upon the first death. We call these annuities \( \bar{A}^{xy}(K) \) with a selectable survivor benefit ratio \( K\%-\text{joint annuities} \). We do not have to explicitly include these annuities in our model, since any survivor benefit ratio \( K \) can be replicated using the three elementary annuities \( \bar{A}^x \), \( \bar{A}^y \), and \( \bar{A}^{xy} \) according to

\[
\bar{A}^{xy}(K) = (1 - K)(\bar{A}^x + \bar{A}^y) + (2K - 1)\bar{A}^{xy}. \tag{2.7}
\]

In turn, this enables us to interpret a given portfolio of \( a \) single annuities for the husband \( \bar{A}^x \), \( b \) single annuities for the wife \( \bar{A}^y \), and \( c \) joint annuities \( \bar{A}^{xy} \) as a portfolio of a \( K\%-\text{joint annuity} \) and residual income from a single annuity. To specify \( K \), we choose to maximize the benefits attributed to the implied \( K\%-\text{joint annuity} \). Hence, if (without loss of generality) we assume that \( a > b \), the portfolio of the three elementary annuities can be reinterpreted as \((2b + c)\) \( K\%-\text{joint annuities} \) \( \bar{A}^{xy}(K) \) with \( K = \frac{b+c}{2b+c} \) and \((a - b)\) single annuities for the husband \( \bar{A}^x \):

\[
a\bar{A}^x + b\bar{A}^y + c\bar{A}^{xy} = b(\bar{A}^x + \bar{A}^y) + c\bar{A}^{xy} + (a - b)\bar{A}^x
\]

\[
= (2b + c) \left\{ \frac{b}{2b+c} (\bar{A}^x + \bar{A}^y) + \frac{c}{2b+c} \bar{A}^{xy} \right\} + (a - b)\bar{A}^x \tag{2.8}
\]

This can be illustrated using a simple example. Let us assume that the husband (wife) receives a single annuity income of 5 (4) and both receive an additional income of 3 from a joint annuity, i.e. \( a = 5, b = 4, c = 3 \). Combining the joint annuity income of 3 with an income of 4 taken from each of the single annuities is equivalent to an income of 11 from an implied \( K\%-\text{joint annuity} \) with \( K = \frac{4+3}{4+4+3} = \frac{7}{11} \approx 64\% \). This leaves the husband (wife) with a residual single annuity income of 1 (0).
2.4 Wealth dynamics

In each period the household has to decide how much of its liquid wealth \(W_t\) to spend on consumption \((C_t)\), life insurance premiums \((LP^x_t, LP^y_t)\) and annuity premiums \((AP^x_t, AP^y_t, AP^{xy}_t)\). Moreover, the household has to choose the fraction of the remaining wealth that will be invested in stocks \((\pi_t)\). Next period’s liquid wealth is given by the remaining wealth including capital market returns, annuity income \((A^x_t, A^y_t, A^{xy}_t)\), and, in case one of the spouses has deceased, payments from life insurance contracts \((L^x_t, L^y_t)\):

\[
W_{t+1} = (W_t - C_t - LP^x_t - LP^y_t - AP^x_t - AP^y_t - AP^{xy}_t) \\
\cdot \left(R_f + \pi_t \cdot (R_{t+1} - R_f)\right) \\
+ A^x_{t+1} + A^y_{t+1} + A^{xy}_{t+1} + (1 - \mathbb{1}^x_{t+1}) L^x_t + (1 - \mathbb{1}^y_{t+1}) L^y_t.
\]

(2.9)

The dynamics of the annuity payments \(A^x, A^y, \text{ and } A^{xy}\) are given by

\[
A^x_{t+1} = \mathbb{1}^x_{t+1} \left(A^x_t + \frac{AP^x_t}{\bar{a}^x_t}\right)
\]

(2.10)

\[
A^y_{t+1} = \mathbb{1}^y_{t+1} \left(A^y_t + \frac{AP^y_t}{\bar{a}^y_t}\right)
\]

(2.11)

\[
A^{xy}_{t+1} = (\mathbb{1}^x_{t+1} + \mathbb{1}^y_{t+1} - \mathbb{1}^x_{t+1} \mathbb{1}^y_{t+1}) \left(A^{xy}_t + \frac{AP^{xy}_t}{\bar{a}^{xy}_t}\right).
\]

(2.12)

We impose that the household is liquidity constraint, such that money cannot be borrowed for financing consumption, insurance products, or stock investments. Furthermore we do not allow short positions in stocks, life insurance, and annuity products:

\[
W_t - C_t - LP^x_t - LP^y_t - AP^x_t - AP^y_t - AP^{xy}_t \geq 0
\]

(2.13)

\[
0 \leq \pi_t \leq 1 \quad LP^x_t \geq 0 \quad LP^y_t \geq 0
\]

(2.14)

\[
AP^x_t \geq 0 \quad AP^y_t \geq 0 \quad AP^{xy}_t \geq 0.
\]

(2.15)

Annuities are illiquid in the sense that the household is prevented from selling annuities. Thus, previously purchased annuity income cannot be reduced and the range of attainable survivor benefit ratios in the embedded K%-joint annuities is restricted to \(0.5 \leq K \leq 1\).

Finally, it is self-evident that we allow for purchases of life insurance and annuities products...
for the living only. So, for example, a widow may still buy single annuities for herself, but no joint annuities.

2.5 Optimization Problem

The household draws utility from consumption according to a time-additive utility function of the constant relative risk aversion type:

\[ u(C_t, s_t) = \frac{1}{1 - \gamma} \left( \frac{C_t}{\phi_{s_t}} \right)^{1-\gamma}. \]  

(2.16)

where \( \gamma \) represents the level of relative risk aversion. Following Love (2010), we normalize the total consumption by the scaling factor \( \phi_{s_t} \), which depends on the family state \( s_t \). This factor can be interpreted as the effective family size. By definition, it is equal to one for a single person household. For a multi-person household it states by which factor joint consumption has to be increased to gain the same utility as a single person. Through shared consumption and economies of scale this factor is expected to be below the number of family members.

The household seeks to maximize expected lifetime utility, expressed recursively through the Bellman equation

\[ J_t(W_t, A_t, s_t) = \max_{C_t, \pi_t, AP_t, LP_t} \left\{ \frac{1}{1 - \gamma} \left( \frac{C_t}{\phi_{s_t}} \right)^{1-\gamma} + \beta \mathbb{E}_t \left[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1}) \right] \right\}, \]  

(2.17)

where \( \beta \) represents the time preference rate. The value function is governed by the state variables liquid wealth \( W_t \), the vector of annuity payments \( A_t = (A^x_t, A^y_t, A^{xy}_t)' \), and family state \( s_t \). The controls are consumption \( C_t \), asset allocation \( \pi_t \), premiums for annuity purchases \( AP_t \) as well as premiums for life insurance purchases \( LP_t \).

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\(^2\) This specification assumes that consumption is equally shared between the spouses, \( C^x = C^y = C/2 \), and that there is jointness in consumption. This is equivalent to the approach by Brown and Poterba (2000) under the assumption of equally weighted, identical subutility functions.
In case both spouses are alive ($s_t = 1$), the expectation in the value function is given by
\[
E_t[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1}) ] = p_t^x p_t^y E_t[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 1) ] \\
+ p_t^x (1 - p_t^y) E_t[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 2) ] \\
+ (1 - p_t^x) p_t^y E_t[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 3) ] \\
+ (1 - p_t^x)(1 - p_t^y) E_t[ J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 4) ].
\] (2.18)

The expectations on the right hand side are conditioned on the family state $s_{t+1}$ and the associated future value functions are only weighted by corresponding survival probabilities. Hence, we do not consider one of the spouses to be a decision maker, who puts more weight on her/his value function. For widowers and widows ($s_t = \{2,3\}$) the expectation only includes one term for the survival of the remaining spouse and one in which the last spouse has deceased, weighted in accordance with the family state transition matrix (2.2). In case both spouses have deceased ($s_t = 4$), the form of the value function depends on the strength of the couple’s bequest motive ($B$):
\[
J_t(W_t, A_t, s_t = 4) = \frac{1}{1 - \gamma} \left( \frac{W_t}{B} \right)^{1-\gamma} \text{ for } B > 0 \\
J_t(W_t, A_t, s_t = 4) = 0 \text{ for } B = 0 .
\] (2.19)

At time $T$ the survival probabilities are zero and the terminal conditions are given by
\[
J_{T+1} = \frac{1}{1 - \gamma} \left( \frac{W_{T+1}}{B} \right)^{1-\gamma} \text{ for } B > 0 \\
J_T = \frac{1}{1 - \gamma} \left( \frac{W_T}{\bar{\phi}_{st}} \right)^{1-\gamma} \text{ for } B = 0 .
\] (2.20)

The optimization problem is homothetic in wealth. Hence, we can decrease the computational effort by normalizing the annuity payments by liquid wealth and thereby reducing the number of continuous state variables. The value function then takes the form
\[
J_t(W_t, A_t, s_t) = (W_t)^{1-\gamma} j(a_t, s_t)
\] (2.21)
with the normalized annuity vector \( a_t = (a_t^x, a_t^y, a_t^{xy})' \). Instead of directly working with the three continuous state variables \((a_t^x, a_t^y, a_t^{xy})'\), we discretize the normalized total amount of annuity payments \( \bar{a}_t = a_t^x + a_t^y + a_t^{xy} \), the share of joint annuities \( a_t^{xy} / \bar{a}_t \), and the husband’s share of single annuities \( a_t^x / (a_t^x + a_t^y) \) on a \( 20 \times 19 \times 19 \) grid. This choice has the advantage that all continuous state variables lie in the interval [0,1] and all their combinations are attainable. While \( a_t^x, a_t^y, \) and \( a_t^{xy} \) lie in [0,1] as well, many combinations result in \( \bar{a}_t > 1 \), i.e. \( A_t > W_t \), which violates the budget restriction (2.9). Hence, our approach avoids unnecessary computational effort while preserving the rectangular grid, which facilitates interpolation.

A three-dimensional grid for annuity payments is only necessary as long as both spouses are alive (\( s_t = 1 \)). For a widow/widower (\( s_t = \{2,3\} \)) it is sufficient to only track total annuity payments, which we discretize on a \( 20 \times 1 \) grid.

On every grid point we solve for the optimal control variables by evaluating the expectation of the future value function using Gaussian quadrature integration over the stock return realizations and cubic spline interpolation.

### 2.6 Model calibration

In our base case calibration, we follow Brown and Poterba (2000) and set the initial age of the husband to 65 and that of the wife to 62, which they argue is a representative calibration for US family structures. Our projection horizon is \( T = 39 \) years, i.e. the maximum age of the husband (wife) is 103 (100). The one-period survival probabilities \((p_t^x, p_t^y)\) are taken from the US 2004 population life table in the National Vital Statistics Report 2007 (Arias 2007) without accounting for mortality trends or cohort effects. We set the coefficient of relative risk aversion to \( \gamma = 5 \), the time-preference rate to \( \beta = 0.96 \), and the bequest motive parameter to \( B = 2 \). These values are commonly used in the life cycle portfolio choice literature (see e.g. Horneff et al. 2009).

For the couple we choose a consumption scaling factor of \( \phi_{s=1} = 1.3 \). This value is in line with the average scaling factor reported in Fernández-Villaverde and Krueger (2007), who
summarize the literature on household equivalence scales. Hence, our value is below the 1.7 used by the OECD (OECD, 1982) but higher than the empirically observed 1.06 in Lazear and Michael (1980) and Nelson (1988).

Following Cocco, Gomes, and Maenhout (2005), the real risk free interest rate is set to 2% \( (R_f = 1.02) \) and the parameters for the lognormal stock distribution are chosen such, that the risk premium is 4% \( (E[R_t] - R_f = 4\%) \) and the stock volatility is 15.7%.

Life insurance and annuity products are priced actuarially fair, using the population mortality table and, in our base case calibration, without any further costs. In line with empirical evidence from the US annuities markets, the maximum age at which annuities can be bought is set to 85. While we are able to model annuities as the illiquid long term investments they are, we have to restrict ourselves to one-year term life insurance contracts for computational reasons. Modeling long term contracts would require two additional (continuous) state variables for tracking life insurance purchases in previous periods. Long term contracts can, however, be qualitatively replicated by repeated purchases of one-year term life insurance, provided that these are available over the complete horizon. Hence, we allow for life insurance purchases up to year \( T - 1 \), although this may provide undue flexibility in adjusting life insurance holdings.

3. Results

3.1 Life Cycle Profiles – Base Case Calibration

In this section we analyze the household’s optimal life cycle behavior. To this end, we present average results of 10,000 simulated life cycles based on previously derived optimal controls. While these controls account for potential changes in the family state, we only present life cycle profiles with both spouses being alive, i.e. we fix the family state to \( s_t = 1 \) in the simulation. This approach ensures that the displayed profiles focus on the decisions of couples and are not mixed with those of singles, whose annuitization behavior is already well documented in the literature.
In what follows, we assume that the couple retires at age 65/62.\textsuperscript{3} Their retirement wealth amounts to 100, for which we distinguish two initial allocations. First, the endowment may consist of liquid wealth only. Alternatively, the endowment may be a combination of liquid wealth and some pre-existing, lifelong pension income for the husband such that the liquid wealth to pension income ratio is 10.\textsuperscript{4} The couple has access to stock and life insurance markets but is subject to alternative restrictions on additional annuity purchases: no access to annuity markets, only access to single annuities, access to single as well as joint annuities.\textsuperscript{5} In combination with the two initial endowment allocations we thus analyze six different settings.

Figure 1 shows the life cycle profile for a couple that is only endowed with liquid wealth and has no access to annuity markets. In the absence of any annuity income the problem is effectively reduced to the Merton (1969) case. Accordingly, the relative amount invested in stocks remains constant over time at $\pi_t = 35.4\%$. Interestingly, the couple does not purchase any life insurance despite their bequest motive. Since there is no alternative to holding liquid wealth for financing future consumption, they automatically hold enough liquid wealth to satisfy their bequest motive. Furthermore, the death of one spouse is not a negative shock – economically. The surviving spouse has to consume less than the couple to draw the same utility, while liquid wealth is unaffected by the death of the spouse. As a consequence, there is no risk for the couple that needs to be hedged by life insurance.

Further simulations, not presented here, that analyze the behavior of a surviving spouse, also show no demand for life insurance and an identical relative stock weight. One reason is that the problems for the couple and for the single spouse are almost identical, i.e. financing consumption without income. A second reason is that stock returns are independent of the survival of the spouses. Therefore, there is no hedging demand with respect to changes in the family status.

Figure 1 here

\textsuperscript{3} In the following we only refer to the husband’s age, keeping in mind that the wife is three years younger.

\textsuperscript{4} With an annuity factor of 13.5 for a 65 year old man, a liquid wealth to pension income ratio of 10 for a total endowment of 100 is achieved by setting annual pension payments to 4.26, which have a present value of 57.4, and liquid wealth to 42.6.

\textsuperscript{5} These restrictions only apply to additional annuity purchases, not to the initial pension income.
As presented in Figure 2, optimal behavior changes notably in case the husband is already endowed with an annuity income. Since this income is comparable to a bond investment, the fraction of liquid wealth invested in stocks is considerably higher than in the former case; well above 70%. In contrast to the previous case, the decease of the husband would result in a negative economic shock for the remaining widow, since all annuity income would be lost. Consequently, there is a strong demand for life insurance on the husband while there is none for the wife. Over the whole life cycle, the face value of the husband’s insurance is just slightly lower than the present value of the husband’s annuity. In case the husband dies, total wealth will decline less than the consumption scaling factor. Hence, the widow can maintain a consumption level that provides about the same utility as before the husband’s death.

*Figure 2 here*

Next, we allow the couple to also purchase single annuities. Figure 3 presents the life cycle profiles in case of a fully liquid initial endowment. Approximately half of the initial wealth is instantaneously annuitized. Between ages 66 and 84 the couple spends small amounts on additional annuities and continuously increases annuity payments. At age 85, the last opportunity to buy annuities, they spend the better part of their remaining liquid wealth on additional annuities. Delaying annuitization is beneficial for two reasons. First, the couple can, on average, earn the equity premium on their liquid wealth, which at younger ages in retirement still exceeds the mortality credit. Second, in case of an early death of one spouse the surviving partner loses less financial wealth.

Looking at the distribution of annuity benefits over the two spouses, we find that comparable income is purchased for both partners, with the wife’s income slightly exceeding the husband’s. This is explained by the difference in life expectancies. The wife is more likely to survive her husband than the other way round. Hence, the couple weighs the widow’s utility more than that of the widower. Compared to the widower, the widow has to finance consumption for a longer period on average, which results in a further increase of annuity demand.

As, prior to age 85, income from the two single annuities is low and the couple retains a substantial amount of liquid wealth, a possible death of one spouse would reduce total
wealth by a factor that is smaller than the couple’s consumption scaling factor. A surviving spouse would be better off economically than the couple was before. At the same time, liquid wealth is high enough to support adequate bequests. Consequently, only a negligible amount of life insurance is purchased for each spouse. This picture changes after the last annuity purchases at age 85. With high annuity incomes at stake and little liquid wealth remaining, the couple decides to increase life insurance purchases.\(^6\) Hence, in case one spouse dies, liquid wealth will be replenished enough to provide for a smooth consumption path as well as for a sufficient bequest later on. In contrast to the case presented in Figure 2, the face values of the two life insurances remain well below the present values of respective annuities. Life insurance prices increase with age, making it too costly to substitute liquid wealth holdings by life insurance over the longer run. Consequently, the couple continues to save from the mid-80s. As liquid wealth increases again, life insurance purchases decline.

Since annuities constitute a major part of the couple’s portfolio over the complete retirement phase, the stock weights are high, between 80% and 100% depending on the remaining amount of liquid wealth.

*Figure 3 here*

Figure 4 shows results for the same setting but with pre-existing annuity income for the husband. Since the husband is already overendowed with annuities, single annuities are purchased for the wife only. For the reasons discussed above, annuity income for the wife is just gradually increased over the first two decades of retirement. After last-minute purchases at age 85, the wife’s income amounts to about 80% of the husband’s benefits.

In line with our findings in Figure 2, the risk of losing the husband’s annuity income is hedged by purchasing life insurance on him with face values comparable to his annuity’s present value. Here, face values decrease faster over time as the wife’s annuity benefits increase and she depends less and less on her husband’s income. Again, no life insurance is purchased on the wife initially. At age 85, when liquid wealth has dropped measurably due to last-minute

\(^6\) In all figures, Panel (A) presents liquid wealth according to Equation 2.9, i.e. already including annuity income received in the respective period. Hence, to determine the liquid wealth a surviving spouse/the heirs would have at hand upon the death of the first/second spouse the deceased spouse’s annuity income has to be subtracted.
annuity purchases, the couple starts to purchase life insurance on the wife as well. In case the wife dies, these endow the widower with enough liquid wealth in order to bequeath.

*Figure 4 here*

Finally we allow the couple to also buy joint annuities. The case with only liquid initial wealth is shown in Figure 5. Similar to the case without joint annuities, the couple sets off by mainly purchasing single annuities and only very little joint annuities. Again the wife’s benefits are slightly higher than the husband’s. In the following years the income of single annuities remains virtually constant and the couple continually increases joint annuity holdings. While benefits from joint annuities do not decrease after the death of one spouse, these annuities are considerably more expensive than single annuities. Consequently, it is reasonable to only gradually increase annuitization, since in case one spouse dies early the remaining spouse can purchase less expensive single annuities.

Along with the purchases of joint annuities the survivor benefit factor of the implied K%-joint annuity rises from an initial 57% to 71% after age 85. Hence, a surviving spouse would receive virtually the same income as in the case without joint annuities, despite the fact that the couple’s total annuity income only comes to 75% of that in the previous case. At the same time, substantially more (bequeathable) liquid wealth remains. Consequently, demand for life insurance is generally low. The better part of the little life insurance holdings is purchased on the wife as a compensation for her slightly higher single annuity income.

Again, we find liquid wealth to be mainly invested in stocks.

*Figure 5 here*

If the husband is endowed with pre-existing annuity income, there is virtually no demand for joint annuities. Apart from negligible purchases of these at age 85, the life cycle profile presented in Figure 6 is identical to the one without access to joint annuities (Figure 4). Due to the husband’s high endowment with single annuities, the couple’s primary concern is again providing for the wife in case the husband dies. This is most effectively achieved by purchasing single annuities for her, as joint annuities are more expensive and the husband does not need additional annuity income.

*Figure 6 here*
3.2 Life Cycle Profiles – Alternative Calibrations

Next, we analyze how our results are affected by introducing loadings on insurance products as well as by choosing alternative calibrations for preference parameters and age difference. As our benchmark, we choose the scenario in which the couple has a fully liquid initial endowment as well as access to single and joint annuities (see Figure 5). We then conduct comparative statics analyses, varying one selected parameter at a time while keeping all other parameters constant. Table 1 presents annuity incomes, the survivor benefit ratio of the implied K%-joint annuity, life insurance face values, liquid wealth holdings as well as the stock fraction at selected ages.

First, we look at the case of actuarially unfair pricing of insurance contracts. In particular, we introduce a loading factor of 10% such that the price of a given annuity income stream or life insurance face value is increased to 110% of the actuarially fair price. The rise in prices drives down the attractiveness of annuities and the couple decides to retain more liquid wealth. The increasing wealth to income ratio leaves the couple less vulnerable to the death of one spouse, which decreases demand for joint annuities and life insurance.

In case the couple is less risk averse ($\gamma = 2$), less funds are annuitized in an effort to cash in on the equity premium. Throughout the life cycle, all liquid funds are invested in stocks. Initially very small, annuity purchases pick up from the mid-70s but even after age 85 the couple holds substantial amounts of liquid wealth. Hence, life insurance purchases are negligible. Highly risk averse couples ($\gamma = 8$), on the other hand, annuitize heavily already early in retirement. With higher risk aversion, optimal controls more strongly account for less probable but unfavorable events, e.g. the widower state. Hence, the difference in annuity income between husband and wife is less pronounced and the couple holds more joint annuities. Again, lower liquid savings lead to higher life insurance purchases.

If there is no jointness in consumption ($\phi = 2$), the death of one spouse reduces consumption needs by 50%. In this case, the couple buys single annuities in equal amounts for husband and wife but virtually no joint annuities. Consequently, annuity income drops by the same factor as effective consumption needs, when one partner deceases. Already right after retirement, the couple purchases the most of their future annuity income. With little income asymmetry and sufficient liquid wealth left, the demand for life insurance is
negligible. If, on the other hand, consumption is perfectly joint ($\phi = 1$), a surviving spouse will have to consume as much as the couple did to draw the same utility. Hence, each spouse is heavily dependent on the overall annuity income and demand for joint annuities is high. This results in an implicit survivor benefit ratio of above 90%, the highest value in all our scenarios. It is, however, well below 100%, as the couple still buys few single annuities. These are cheaper than joint annuities and allow the impatient couple to increase annuity income today at the price of slightly lower income for the surviving spouse. Liquid wealth holdings as well as life insurance purchases are similar to those in the base case, particularly later in retirement.

In case the couple has no bequest motive, the couple annuitizes more of its liquid wealth than in the base case, especially in their 70s. From age 85 on, liquid wealth is virtually depleted and only consists of current annuity income. When having a high bequest motive, the couple has to hold more liquid wealth and overall annuity income is lower than in the base case. A substantial fraction of liquid wealth is reserved for bequest purposes and consumption has to be financed primarily through annuity income. Even after the death of one spouse and the resulting drop in annuity income, the surviving spouse cannot heavily consume from liquid wealth. Hence, purchases of joint annuities and the survivor benefit ratio increase, as do purchases of life insurance to provide at least some disposable liquid wealth for the surviving spouse.

In our model, husband and wife merely differ in terms of their respective mortality rates. Equal mortality rates would result in symmetric annuitization and life insurance purchase behavior. In our base case, the wife’s mortality rates are lower due to the usual gender difference as well as the assumed age difference of 3 years. Her single annuity and life insurance holdings are higher than those of the husband. Setting the age gap to zero and only retaining the gender specific asymmetry in mortality rates decreases these differences. Overall annuity income increases as the wife’s annuities become cheaper. In analogy, with an age difference of 10 years, the wife’s single annuity income is increased at the cost of both her husband’s as well as joint annuity income and the couple again holds more life insurance on her.

Overall, it is noticeable that for most parameterizations targeted survivor benefit ratios of the implied K%-joint annuities remain at about 70%. Only for variations in consumption
scaling the survivor benefit ratios differ substantially. The next section will study this effect in more detail.

### 3.3 Optimal Survivor Benefit Ratio

For an analysis of the impact of consumption scaling on optimal survivor benefit ratios we again turn to our base case couple from Section 3.1, which has a fully liquid endowment with retirement savings of 100 as well as access to bonds, stocks and life insurance products. In contrast to previous analyses, we only allow for annuity purchases at age 65. This enables us to uniquely determine an optimal ratio that is neither dynamic nor path dependent.

Table 2 presents survivor benefit ratios, annuity incomes, and annuity expenditures for consumption scaling factors ranging from $\phi = 1$ (complete jointness of consumption) to $\phi = 2$ (no scaling effects). As already indicated in the previous section the optimal survivor benefit ratio decreases with increasing consumption scaling factor. In fact, it almost matches the inverse of the consumption scaling factor. This is a plausible result, since the surviving spouse draws the same utility as the couple did before when reducing consumption by the inverse of the scaling factor. To reduce the survivor benefit ratio the couple substitutes more and more joint annuities by single annuities. A couple with scaling factor $\phi = 1$, for example, only purchases joint annuities and receives an annual income of 3.7. By contrast, a couple with a scaling factor of $\phi = 2$ has an annuity income of 4.75, consisting of 2.32 (2.37) from the husband’s (wife’s) single annuity and only a negligible 0.06 from joint annuities. For all scaling factors, the difference between the husband’s and the wife’s annuity income is very low. With a maximum of 0.05, it is substantially smaller than the difference in our base case scenario with dynamic annuitization. Consequently, almost the whole annuity income (at least 98.7%) can be attributed to a K%-joint annuity, in line with section 2.3.

Independent of the consumption scaling factor, the overall demand for annuities is high. Expenditures on annuities vary between 70.7 to 72.5 percent of the initial endowment. Considering the substantial differences in the couples’ preferences, this is a surprisingly stable result. The demand for life insurance is non-existent, since neither does the small difference in the spouses’ annuity incomes have to be compensated nor does the remaining liquid wealth have to be replenished in order to provide appropriated bequests.
Table 2 here

3.4 Welfare Analysis

So far, we have seen that depending on the level of initial annuitization there may be substantial demand for annuities and life insurance. In this section we now study whether demand for these products also coincides with a substantial increase in the couple’s welfare. To this end, we calculate certainty equivalents of the utility at age 65, defined as the inverse contribution function of the utility:

$$CE_t(W_t, A_t, s_t) = \left( (1 - \gamma) \cdot J_t(W_t, A_t, s_t) \right)^{\frac{1}{1-\gamma}}$$

Using this measure, we evaluate alternative combinations of initial wealth annuitization and availability of annuities, life insurance, and stocks. For the reason of comparability, in all these settings the couple is endowed with the same amount of total wealth and has baseline preference factors for risk aversion ($\gamma = 5$), time preference ($\beta = 0.96$), consumption scaling ($\phi_{s=1} = 1.3$), and bequest motive strength ($B = 2$).

Regarding the level of initial annuitization we distinguish three cases: no annuitization, single annuitization, and 67%-joint annuitization. The first two are already known from the life cycle profiles in section 3.1: in no annuitization initial wealth is completely liquid and in single annuitization initial wealth is partly annuitized in a single annuity for the husband, such that the liquid wealth to income ratio is 10. The present value of this annuity corresponds to 57.4% of the initial total wealth. In the 67%-joint annuitization case the couple owns a K%-joint annuity with a survivor benefit ratio of $K = 0.67$. Again, the liquid wealth to income ratio is 10, which corresponds to 62.3% of initial total wealth being annuitized.
Results are presented in Table 3. As the absolute values of the certainty equivalents depend on the arbitrary level of total wealth, we normalize all values by the certainty equivalent of the case, which we already used as benchmark in section 3.2, i.e. no annuitization with access to stocks, life insurance and to all annuities until age 85 (Column A, Row 1 in Table 3).

Table 3 here

Panel I of Table 3 shows the effects of annuity availability in case the couple has access to stocks and life insurance. When being able to optimally purchase additional annuities, the couple loses welfare in case some of its initial wealth is already pre-annuitized (Row 1). The highly asymmetric distribution of annuity income in the single annuitization case reduces welfare by 2.1% (B1). For the more symmetric 67%-joint annuitization, on the other hand, welfare losses only amount to 0.1%, indicating that the pre-set annuity distribution is similar to the couple’s optimal annuitization decision (C1).

For no annuitization, the difference in the certainty equivalent for joint annuities being available (A1) or not (A2) is only very small (about 0.1%), since with the combination of life insurance and delaying annuitization a surviving spouse can be provided with enough wealth, such that a mortality shock decreases welfare only marginally. For single annuitization there is only demand for single annuities for the wife (see Figure 6 in section 3.1). Consequently the availability of joint annuities does not enhance welfare (B1 vs. B2). In the case of 67%-joint annuitization the couple is already endowed with sufficient joint annuities and the couple does not appreciate the possibility to buy additional ones (C1 vs. C2).

Independent of the level of initial annuitization, being allowed to buy annuities up to age 102 instead of 85 only slightly increases welfare (Row 3). By age 85, the couple is already heavily annuitized and, therefore, loosening this restriction only has little impact on consumption for very advanced ages. In addition, utility from those years is heavily discounted due to increasing mortality and time preference and, hence, the influence on the certainty equivalent at age 65 is negligible.

If the couple is not initially endowed with annuities and can purchase them only at the beginning of retirement, welfare losses amount to 1.4% (A4 vs. A1). In this case, additionally lacking the access to joint annuities is much more severe with the further decrease in
certainty equivalent amounting to 1.7% (A5 vs. A4 compared to A2 vs. A1). Not being able to either delay annuitization or purchase joint annuities, the couple has to buy a lot of single annuities immediately and, hence, is heavily exposed to the risk of early income losses. With welfare gains of 4.4% the option to buy annuities until age 85 is of greater value to the couple in the single annuitization case (B4 vs. B1). If the husband dies early, the widow can still buy additional annuities from the payouts of the husband’s life insurance. As discussed above (B2 vs. B1), joint annuities are not welfare increasing in this setting (B5 vs. B4).

For 67%-joint annuitization the restriction to buy annuities only at the beginning of retirement decreases welfare by the same amount as for no annuitization (C4 vs. C1 compared to A4 vs. A1). Not having access to joint annuities, however, leads to welfare losses of only 0.3% (C5 vs. C4), since the couple is already endowed with some joint annuities.

If the initial wealth is not annuitized and the couple is not allowed to buy annuities at all, the welfare dramatically decreases by 17.2% in comparison to the benchmark case (A6 vs. A1). In other words, the couple would be willing to surrender 17.2% of its initial retirement wealth to get access to the annuity market, which emphasizes the importance of this asset. The welfare losses only amount to 12.8%, if the husband is initially endowed with a single annuity (B6 vs. B1). For 67%-joint annuitization the welfare loss is substantially reduced to 2.1% (C6 vs. C1), reflecting that this initial annuitization is close to the optimal one-time choice derived in section 3.3.

Interestingly, the welfare loss of 17.2% (A1 vs. A6) for a couple having no access to annuity markets is small compared to an individual living on his own. An equivalent calculation (not shown in Table 3) for a single man at age 65 with only liquid wealth reveals that the restriction to be not allowed to buy annuities results in a much higher welfare loss of 32.5%. According to Kotlikoff and Spivak (1981), the pooling of mortality risk in a couple’s household is comparable to an incomplete annuity market. The probability that a single individual reaches very high ages and outlives his assets is considerably higher than the probability of both spouses reaching these ages. Additionally, an early death of one spouse would leave the surviving spouse with more assets guaranteeing consumption even up to high ages. As a consequence, the couple’s mortality risk is partially hedged, such that annuities are of less importance to them compared to an individual.
Panel II of Table 3 presents welfare implications of varying availability of annuities, if the couple does not have access to the stock market. Comparing A7 and C7 to our base case in A1, we find that being prevented from investing in stocks results in a decrease of certainty equivalent of 6.5%. Even though a large fraction of wealth is annuitized, it is still important for the couple to be able to invest the remaining liquid wealth in stocks. Compared to the respective no annuitization cases, welfare losses due to single annuitization are lower if the couple does not have access to stocks (B7 vs. A7 compared to B1 vs. A1). Without stocks the couple’s annuity purchases increase substantially. Hence, the couple is much less overendowed with single annuities for the husband and can get closer to their optimal annuitization choice.

Without having the opportunity to benefit from the equity premium, the couple not only purchases more annuities but also purchases them earlier. Consequently, the welfare losses of only allowing annuitization at age 65 are smaller than in the settings with available stocks (Row 9 vs. Row 7 compared to Row 4 vs. Row 1). At the same time, completely denying access to annuity markets results in even higher welfare reductions (Row 11 vs. Row 7 compared to Row 6 vs. Row 1).

Comparing A6 and A7 to A1, we find that for couples with in the no annuitization case losing access to annuities is much more welfare reducing than losing access to the stock market, negative 17.2% vs. negative 6.5%. By contrast, in case the couple is initially endowed with a 67%-joint annuity, being withheld from purchasing additional annuities results in less severe welfare losses than not being able to participate in the stock market, negative 2.1% vs. negative 6.4% (comparing C6 and C7 to C1). Hence, which asset class is the most important for the couple strongly depends on the couple’s situation.

Panel III of Table 3 presents welfare implications of varying availability of annuities in the couple does not have access to life insurance. From section 3.1 we now that the demand for life insurance is very low if the couple has the opportunity to symmetrically annuitize for both spouses. Hence, the welfare losses from not being able to purchase life insurance are negligible for all cases of no annuitization and 67%-joint annuitization (Panel III vs. Panel I for Columns A and C). Here, the only small exception is A15 with a loss of certainty equivalent of 0.9%. For this case without joint annuities and no option to delay annuitization, life insurance would help to protect against the financial loss due to an early
death of one spouse. By contrast, in case the couple is annuitized highly asymmetrically, i.e. single annuitization, not having access to life insurance results in dramatic welfare losses (Panel III vs. Panel I for Columns B). In fact, the loss of access to life insurance is more severe than the being withheld from purchasing additional annuities, negative 14.2% vs. negative 12.8% (comparing B12 and B6 to B1).

Finally, Panel IV presents results for the cases in which the couple neither has access to stocks nor to life insurance. Comparing A19 to A1, we find that a couple that is only allowed to invested in risk-free bonds suffers substantial welfare losses of 24.7% compared to our base case. This case might also be interpreted as a financially illiterate couple that does not make use of the capital and insurance markets. For such a couple, making a one-time optimal annuitization decision at retirement reduces welfare losses by more than two-thirds to 6.9% (A18 vs. A19). If, on the other hand, a financially illiterate couple already holds substantial amounts of annuities, welfare losses only amount to 9.7% (C19 vs. C1) and making an additional one-time annuitization decision only reduces welfare losses to 6.8% (C18 vs. C1).

4. Conclusion

In this paper, we present a life cycle asset allocation model for a couple in retirement with access to the markets for stocks, annuities, and life insurance. For this couple, uncertainty about their joint lifetimes is the major risk. This risk has two aspects. On the one hand, the couple might outlive their assets. On the other hand, an early death of one spouse might deprive the surviving partner of highly valued annuity income. Our paper aims at investigating how the couple can optimally hedge these risks by dynamically investing into annuities, life insurance, stocks and bonds. Here, we particularly focus on the distribution of annuitized funds over single and joint annuities.

In accordance with Brown and Poterba (2000) we come to the conclusion that K%-joint annuities are highly welfare increasing. These products are very appealing to the couple, since they hedge both risks simultaneously by guaranteeing a lifelong income stream corresponding to the couple’s varying consumption needs. The optimal survivor benefit ratio of these annuities is tightly linked to the level of scaling of the couple’s consumption relative
to that of a single consumer. Moreover, most of the welfare gains can be achieved by a one-time optimal annuitization decision at the beginning of retirement.

We show that term life insurance is an effective product to insure survival contingent annuity income of one spouse against an early death. Particularly in case a large fraction of wealth is annuitized for only one spouse, life insurance contracts enhance welfare substantially. These are also the only cases in which life insurance is used to support bequest. Otherwise, liquid wealth is preferred over life insurance to finance bequest.

An important aspect not accounted for in our analysis is the modeling of health risk. French (2005), Yogo (2009), and Pang and Warshawsky (2010) include uncertain health status in their life cycle models and show a significant impact on the degree of annuitization. Another possible extension is the inclusion of taxes. In case life insurance provides an effective tax shelter, these contracts might be more appealing for financing bequests.

References


Table 1: Parameter Variations

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<th>Base Case</th>
<th>Loadings = 10%</th>
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<td>Age</td>
<td>Age</td>
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<td>75</td>
</tr>
<tr>
<td>Ann. Income – Husband</td>
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<td>Ann. Income – Wife</td>
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<td>Ann. Income – Joint</td>
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<td>Survivor Benefit Ratio</td>
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<th>High Risk Aversion (γ = 8)</th>
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<tr>
<td>Age</td>
<td>Age</td>
</tr>
<tr>
<td>66</td>
<td>75</td>
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<tr>
<td>Ann. Income – Husband</td>
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<table>
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<th>No Consumption Scaling (𝜙 = 2)</th>
<th>High Consumption Scaling (𝜙 = 1)</th>
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<tr>
<td>Age</td>
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Notes: This table shows the effect of parameter variations on the couple’s life cycle profiles, represented by the values for income of both single as well as joint annuities, the implied survivor benefit ratio, life insurance face values for both spouses, liquid wealth holdings, and chosen stock fraction of liquid savings at certain ages. In all cases the couple starts with only liquid wealth of 100 at age 65 and may invest in stocks and bonds, purchase life insurance and single as well as joint annuities (up to age 85). Each case varies a certain aspect or preference parameter in comparison to the base case scenario, which has no loading factors on annuities and life insurance products, relative risk aversion γ = 5, consumption scaling 𝜙 = 1.3, bequest motive of 𝐵 = 2, and an age difference for husband and wife of 3 years.
Table 2: Dependence of survivor benefit ratio on the consumption scaling

<table>
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<tr>
<th>Scaling Factor $\phi$</th>
<th>$1/\phi$</th>
<th>Survivor Benefit Ratio $K$</th>
<th>Annuity Income</th>
<th>Expenditure on Annuities</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Husband $A^x$</td>
<td>Wife $A^y$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.00</td>
<td>1.00</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1.1</td>
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<td>0.77</td>
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<td>0.93</td>
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<td>1.18</td>
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<td>0.64</td>
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<td>1.64</td>
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<td>0.51</td>
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<td>2.37</td>
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Notes: This table shows the optimal annuitization decision in dependence of the couple’s consumption scaling factor $\phi$ (Column 1, its inverse in Col. 2), if their initial wealth of 100 is not pre-annuitized and they are only allowed to purchase annuities once at the age 65. Presented are: the survivor benefit ratio of the implicit $K\%$-joint annuity (Col. 3), the incomes of the purchased single and joint annuities (Col. 4 to 6), and the total expenditure on annuities (Col. 7). In each case the couple has a parameter of risk aversion $\gamma = 5$, time preference rate $\beta = 0.96$, and parameter for bequest $B = 2$. 
Table 3: Certainty equivalents at age 65 for different settings of pre-annuitized initial wealth and available financial products

<table>
<thead>
<tr>
<th>Annuity Purchase Restrictions</th>
<th>Pre-Annuitization Settings</th>
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<td>No Annuitization (A)</td>
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<td>Annuity Types Available until Age</td>
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<td>Annuity Types Available until Age</td>
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<tr>
<td>Panel I: Life Insurance and Stocks Available</td>
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<tr>
<td>(1) Single &amp; Joint 85</td>
<td>1.000</td>
</tr>
<tr>
<td>(2) Single 85</td>
<td>0.999</td>
</tr>
<tr>
<td>(3) Single &amp; Joint 102</td>
<td>1.001</td>
</tr>
<tr>
<td>(4) Single &amp; Joint 65</td>
<td>0.986</td>
</tr>
<tr>
<td>(5) Single 65</td>
<td>0.969</td>
</tr>
<tr>
<td>(6) None</td>
<td>-</td>
</tr>
<tr>
<td>Panel II: Life Insurance Available; No Stocks</td>
<td></td>
</tr>
<tr>
<td>(7) Single &amp; Joint 85</td>
<td>0.935</td>
</tr>
<tr>
<td>(8) Single 85</td>
<td>0.933</td>
</tr>
<tr>
<td>(9) Single &amp; Joint 65</td>
<td>0.933</td>
</tr>
<tr>
<td>(10) Single 65</td>
<td>0.921</td>
</tr>
<tr>
<td>(11) None</td>
<td>-</td>
</tr>
<tr>
<td>Panel III: No Life Insurance; Stocks Available</td>
<td></td>
</tr>
<tr>
<td>(12) Single &amp; Joint 85</td>
<td>0.999</td>
</tr>
<tr>
<td>(13) Single 85</td>
<td>0.996</td>
</tr>
<tr>
<td>(14) Single &amp; Joint 65</td>
<td>0.986</td>
</tr>
<tr>
<td>(15) Single 65</td>
<td>0.960</td>
</tr>
<tr>
<td>(16) None</td>
<td>-</td>
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<tr>
<td>Panel IV: No Life Insurance; No Stocks</td>
<td></td>
</tr>
<tr>
<td>(17) Single &amp; Joint 85</td>
<td>0.931</td>
</tr>
<tr>
<td>(18) Single &amp; Joint 65</td>
<td>0.931</td>
</tr>
<tr>
<td>(19) None</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: This table shows the certainty equivalent at the (husband’s) age 65 for different settings. The couple has a level of risk aversion $\gamma = 5$, time preference rate $\beta = 0.96$, consumption scaling factor $\phi_s = 1.3$, and parameter for bequest $B = 2$. In all settings the initial wealth is equal, but they differ with respect to the level of pre-annuitization and the availability of financial products. For the pre-annuitization there are three cases: (A) no annuitization with only liquid wealth, (B) single annuitization with a single annuity for the husband, and (C) 67%-joint annuitization with a K%-joint annuity with a survivor benefit ratio $K = 0.67$. In cases (B) and (C) the amount of annuitization is such that the liquid wealth to income ratio is 10. Panels I to IV differ with respect to the availability of stocks and term life insurance contracts. For the particular cases either single and joint annuities, only single annuities, or no annuities at all are available. In each case there is a maximum age, after which additional annuities cannot be purchased. All certainty equivalents are normalized by the value for the base case (A1). Reading example: the certainty equivalent of (B11) is only 77.4% of the certainty equivalent of the base case. This couple would forego up to 22.6% of its initial wealth to have the same initial annuitization and product availability as in the base case.
Figure 1: Expected life cycle profiles for couples with no pre-annuitized wealth with access to bonds, stocks and life insurance, but no access to (additional) annuities.


Notes: The couple has a level of risk aversion $\gamma = 5$, time preference rate $\beta = 0.96$, consumption scaling factor $\phi_s = 1.3$, and parameter for bequest $B = 2$. Insurance contracts are actuarially fair priced. We use optimal feedback controls obtained from the stochastic optimization for a couple with maximum lifespan of (husband’s) age 102; expectations are computed from 10,000 Monte Carlo simulations conditional on the survival of both spouses with initial total wealth set to 100.

Figure 2: Expected life cycle profiles for couples with partly annuitized wealth (single annuity for the husband with liquid wealth to income ratio 10) with access to bonds, stocks and life insurance, but no access to (additional) annuities.


Notes: see figure 1.
Panel (A) Wealth  Panel (B) Life Insurance  Panel (C) Annuities  Panel (D) Stock Weight

Figure 3: Expected life cycle profiles for couples with no pre-annuitized wealth with access to bonds, stocks, life insurance and single annuities (up to age 85), but not to joint annuities.


Notes: see figure 1

Panel (A) Wealth  Panel (B) Life Insurance  Panel (C) Annuities  Panel (D) Stock Weight

Figure 4: Expected life cycle profiles for couples with partly annuitized wealth (single annuity for the husband with liquid wealth to income ratio 10) with access to bonds, stocks, life insurance and single annuities (up to age 85), but not to joint annuities.


Notes: see figure 1
Figure 5: Expected life cycle profiles for couples with no pre-annuitized wealth with access to bonds, stocks, life insurance and single as well as joint annuities (up to age 85).


Notes: see figure 1
Figure 6: Expected life cycle profiles for couples with partly annuitized wealth (single annuity for the husband with liquid wealth to income ratio 10) wealth with access to bonds, stocks, life insurance and single as well as joint annuities (up to age 85).


Notes: see figure 1