Automatic adjustment mechanisms and budget balancing of pension schemes

Frédéric Gannon (U. Le Havre)
Florence Legros (U. Dauphine)
Vincent Touzé (Sciences Po - OFCE)

February 2013
Very preliminary version - Comments welcome

This article is dedicated to the study of the Automatic Adjustment Mechanims (AAM) used in some pension systems. The objective of the paper is twofold. First, we identify different types of automatic adjustment rules adopted (or which could be adopted) by the main developed countries and we show how these adjustments contribute to better solvency. Unfortunately, these adjustments are not sufficient to guarantee an intertemporal balanced budget. That leads us to discuss the opportunity to use Automatic Balance Mechanisms (ABM). Second, we build an example of ABM which would result from an optimal tradeoff between increasing the receipts and reducing the expenditures. The ABMs obtain from the minimizing of a cost function and ensure an intertemporal budgetary balance. We then apply this rule to American and French cases to evaluate the adjustments necessary to ensure the financial solvency. These assessments are conducted under different assumptions on forecast time horizon, time preference and weighting of social costs associated with increased revenue or lower expenditure.

In general, governments are reluctant to reform pension systems because they fear that the reforms might induce too high political costs. As a consequence, they tend to procrastinate and to postpone the adoption of measures that would guarantee solvency. Of course, faced with the emergency of the insolvency of their pension systems, all governments have conducted reforms - some of them very deep - but without setting restoring forces. The problem with *ad hoc* reforms is that to quote Turner (2009), "(they) have a high degree of political risk because their timing and magnitude are unknown".

To avoid that the future states of the pension system depend on choices that politicians would not take willingly, two types of strategies are used by governments: first, delegate the management of the pension systems to competent independent authorities. Second, introduce new rules to allow for automatic adjustment mechanisms (AAM). These AAM would guarantee the solvency of the system at any date without needing political intervention and eliminating the "need for large program changes made in crisis mode" (Turner, 2009).

This means straightforward and clear choices about transfers between generations and a strong underlying social acceptance. To avoid regular and politically costly *ad hoc* reforms, Sweden has first adopted Notional Defined Contributions (NDC) plans in 1994. Then, to reinforce the robustness of the system, it has launched an Automatic Balance Mechanism (ABM) that relies on the key rule that every year, the solvency of the pension system must be checked, thanks to the flexibility of the present and future pension benefits (Settergren, 2001). The return of the "savings" invested in the NDC crucially depends on this indexation. In 1983, the U.S. government has chosen a radical long run reform (mainly by increasing the payroll taxes and raising the full pension age) which potentially guarantees an intertemporal balanced budget for about half a century. Debt emission is not allowed for the U.S. Social Security. This constraint plays the role of a strong restoring force, since it forces to plan surpluses to compensate anticipated deficits. In France, the political debate about pensions issue is often a source of conflicts (Blanchet and Legros, 2002). Important reforms have been adopted in 1993, 2003 and 2007. To pacify the debate, an authority composed of experts and social partners (Conseil d'orientation des retraites) was created in 2001. Its function is only advisory but its large freedom of investigation along with its published studies, allows it to contribute to a better literacy and social consensus, which can facilitate the political decisions and reduce the populist temptations.

Let us turn now to the related literature on automatic adjustment mechanisms. Turner (2009) defines the concept of automatic adjustments and give some examples of its applications to twelve high-income countries, categorized in five groups. First, countries with traditional pay-as-you-go (PAYG) systems with life expectancy indexing (LEI) of pension benefits: Portugal, Finland, Norway. Second, countries that have systems with both NDC and

LEI of benefits: Italy and Poland. Third, countries using LEI of the earliest age at which social security benefits can be received: United Kingdom, Denmark. Fourth, countries with AAM that are tied to solvency: Sweden, Germany, Japan, Canada. Fifth, countries that automatically adjust other parameters of their social security systems, such as the years of contributions required for a full benefit: France.

Bosworth and Weaver (2011) focus on the social security autopilot by analyzing the contrasted cases of AAM in Canada, Sweden, Germany and Italy, to assess their possible implementation in the U.S. They recall that if semi-automatic stabilizing mechanisms have been used in the U.S. for the last forty years, true automatic stabilizing mechanisms (ASM) are more recent.

Shoyen and Stamati (2011) concentrate on the peculiar role of autopilot in the NDC with an application to Sweden and Italy.

Solvency issues have been investigated by Vidal-Melia and Boado-Penas (2010). They precise the connection between the contribution asset and the hidden asset (similar to the equivalent concepts of "hidden tax", "implicit tax on pensions" or "PAYG asset" used in the literature) to evaluate whether using either of these to compile the actuarial balance in PAYG pension systems would provide a reliable solvency indicator. The contribution asset can be interpreted as the maximum level of liabilities that can be financed by the existing contribution rate without periodic supplements from the sponsor, *ceteris paribus*. The hidden asset is the present expected value of the hidden - or implicit - taxes that the system will apply to its participants in the future, defined as contributions in excess of those that would be needed by a capitalized system to pay the same benefits. The authors scrutinize the Swedish "actuarial balance" to identify the elements characterizing these two concepts. They find that only the contribution asset is applied. That leads them to qualify the hidden asset as only a theoretical device, mainly because its computation requires projections of economic, demographic and financial variables. However, interestingly enough, the U.S. administration [1] defines the hidden asset used to assess the actuarial balance as the system's "unfunded obligation" with a perpetual horizon, or, explicitly, "the present value of future benefits less future contributions for current and future participants, considering that the reserve fund dry and that legislation is constant".

Capretta (2006) considers the examples of Sweden and Germany to address the issue of automatic solvency of U.S. social security. He stresses that, although correcting for longer life spans helps stabilize costs, it is not

---

[1] Board of Trustees, Federal Old-Age and Survivors and Disability Insurance Trust Funds (BOT), 2009, 2008 Annual Report

sufficient to assure solvency at a fixed contribution rate, as fertility and population growth, labor force participation patterns, and productivity growth all play important roles in long term pay-as-you-go financing. As a result, Sweden adopted an ABM, whereas Germany links annual pension indexing to changes in the ratio of pensioners to workers supporting the system, the so-called "sustainability factor". He recalls the suggestion by Steuerle and Penner (2005) to start the process of automatic adjustments in the U.S. social security by setting the normal retirement age administratively, taking into account the increase of life expectancy, hence mimicking the Swedish NDC's annuity divisor device. However, according to Capretta, it may be easier for the U.S. to adopt an adjustment factor similar to Germany's "sustainability factor". He insists that Congress would be more likely to adopt a mechanistic provision that guarantees to future generations of retirees the same number of years, on average, in benefits as the current generation - automatically.

The aim of this paper is to characterize the properties of the AAM and to propose an ABM. Our analysis shows how the AAM can contribute to a better solvency (ASM). We clarify the principle of Automatic Balance Mechanism. That requires to define a measure of the intertemporal budget balance (for example U.S. actuarial balance or Sweden balance ratio) and to fix the time horizon. First section defines the intertemporal pension budget constraint. The second section addresses the issue of AAM: what are their roles in adjusting, stabilizing and balancing? In the third section, we build a "smooth" ABM, assuming a trade-off between present and future receipts and expenditures. The third section is dedicated to an application of this ABM to U.S. Social Security and French defined benefit system (CNAV). The last section concludes.

The intertemporal budget balance of the pension system writes:

$$R \cdot F^{-} + \tau \cdot \sum_a w_a \cdot e_a \cdot N_a = \sum_a p_a \cdot \lambda_a \cdot N_a + F \tag{1}$$

where R is the interest factor, τ the contribution rate, F the value of the financial asset (reserve fund) and at each age a, w_a is the average wage, e_a the employment rate[2], N_a the size of population, p_a the average pension and

---

[2] In practice, $e_a = (1 - u_a) \cdot (1 - \lambda_a - i_a)$ where u_a is the rate of unemployment and i_a the inactivity rate of pensioners. If we suppose rationed job market by a work demand $\bar{L}_a$, the unemployment rate is an adjustment variable:
$u_a = \frac{(1 - \lambda_a - i_a) \cdot L_a - \bar{L}_a}{(1 - \lambda_a - i_a) \cdot L_a}$.In this case, there is a conflict (a tradeoff) between rate of retire-

λ_a the percentage of retired people. In practice p and d are influenced by the legislation. We suppose that these two parameters and τ can be controlled by the social planner.
The dynamics of the population is driven by a simple equation:

$$N_a = q_a \cdot N_a^- \tag{2}$$

where q_a is the survival rate.
We denote Γ the payroll growth rate: $\Gamma = \frac{\sum_a w_a \cdot e_a \cdot N_a}{\sum_a w_a^- \cdot e_a^- \cdot N_a^-}$. $w = \frac{\sum_a w_a \cdot e_a \cdot N_a}{\sum_a e_a \cdot N_a}$ is the average wage. $p = \frac{\sum_a \lambda_a \cdot d_a \cdot N_a}{\sum_a d_a \cdot N_a}$ is the average pension. $d = \frac{\sum_a \lambda_a \cdot N_a}{\sum_a e_a \cdot N_a}$ is the dependency ratio. $\rho = \frac{p}{w}$ is the global replacement rate.

The intertemporal balance writes:

$$\begin{aligned} \frac{R}{\Gamma} \cdot f^- + \tau &= \frac{\sum_a p_a \cdot \lambda_a \cdot N_a}{\sum_a w_a \cdot e_a \cdot N_a} + f \\ &\Leftrightarrow \\ \frac{R}{\Gamma} \cdot f^- + \tau &= \frac{\sum_a p_a \cdot \lambda_a \cdot N_a}{\bar{w} \cdot \sum_a e_a \cdot N_a} + f \\ &\Leftrightarrow \\ \frac{R}{\Gamma} \cdot f^- + \tau &= \rho \cdot d + f. \end{aligned} \tag{3}$$

What about solvency (Boado-Penas, Vidal-Meliá and Sakamoto, 2010)? From an accountancy point of view, there are different methods to estimate the implicit liabilities and the solvency of unfunded pension systems. In practice, two measures of solvency are used. The first is an assessment of the discounted sum of revenues and expenditures. This valuation approach is used in the United States to assess the present value of the underfunding of the system. Sweden has opted for another method (Settergren, 2001). It defines its pension plan is solvent when:

> Present value of contributions payable by workers alive today
> + Value of the reserve fund
> =
> Value of pension commitments towards generations alive today.

The general problem of the social planner or the government is how to adjust parameters (τ, p_a, d_a) with time. Adopting automatic adjusment rules means that a law of motion of parameters as function of economic/demographic variables is chosen.

---

ment and unemployment rate.

With the AAM, the institutional parameters are adjusted according to the predefined rules. Otherwise, the changes are considered as discretionary decisions. They are therefore subject to the hazards of political choice.

Choosing a specific Automatic Adjustment Mechanism requires to define several elements (see Bosworth and Weaver, 2011):

- Legitimating the rules according to the example "one objective, one tool." We need to identify objectives and tools (parameters). Main objectives concern equity, social justice and solvency.

- Defining the frequency of review.

- Choosing the elements that will base the adjustment.

- Setting adjustments as *ex ante* based on expectations (prediction-based) or *ex post* based on the states of nature.

- Fixing the degree of automaticity (no questioning): the adjustments are mandatory, which guarantees credibility of the process.

Turner (2009) shows how in practice many parameters (pension, eligibility age, etc) are indexed on changes in the life expectancy, consumer price index or wage growth, etc. In general, these adjustments allow to reduce the gap between receipts and expenditures, but there are not sufficient enough to guarantee a fully balanced budget.

For each retired people i, the pension benefit evolves each year as follows: $p_i = I \cdot p_i^-$ where I is an indexing factor. The main objective of the indexing factor is to preserve the level of quality of life. In general, I corrects the increase in the cost of living and permits to maintain the purchase power of the pension. In the past (in some countries), I was equal to the factor of wage growth. In this case, the indexing coefficient allows to maintain the relative purchase power between workers and pensioners. Generally, indexing on the average wage benefits to pensioners more than indexing on the consumer price. Also, this indexation has a positive effect on the solvency because the gap between current wages and pensions increases with time. However, the economic crisis may imply that nominal wages growth be lower than inflation.

To obtain a full pension requires to validate a sufficient number of quarters. The duration of the assessment period can be connected to life expectancy. The underlying idea is to guarantee to each generation a similar ratio: time to retire / time worked.

In France, the 2003 law induced an automatic revision of the contributory period per generation with respect to changes in the life expectancy.

In many countries, the reforms have modified the law relative to the legal retirement ages. The minimum age is the age at which workers can liquidate their pensions (for example, 61 y.o in Sweden). The normal retirement age is the age which serves as a reference to define he full pension. Generally, the adjustment is not automatic but planned by the law. In practice, with a given frequency, these ages could be revised w.r.t new informations about changes in life expectancy for each cohort.

In Sweden the coefficient of conversion of capital into an annuity depends on the age and the birth year. This coefficient is revised to reflect the evolution of generation mortality tables.

In France, the 1993 reform planned changes in the pension-earnings links by increasing the reference period to compute the life cycle average wage which is used in the first pillar pension scheme to determine the benefit. It is not an automatic adjustment but a planned one. This correction has gradually reduced the pensions of the new generations to retire, which could be interpreted as taking into account the increase in life expectancy at retirement.

What happens if AAM are not sufficiently stabilizing In 2001, Sweden opted for type of ABM. The choice of an ABM raises four major issues:

- How is defined the pension budget balance?
- What are the criteria for choosing changes in current law?
- What room is left for optimization?

- What planning time horizon for full balancing? The difficulty is to define a reference horizon and the frequency of the automatic adjustments.

As the AAM, the Automatic Balance Mechanisms can be determined:

- Ex ante: shocks are anticipated and changes in law are planned.

- Ex post: the law evolves w.r.t to the knowledge of the states of nature. Changes concern the pension formula parameters and the contribution rate.

$$f = 0$$

- If d_a and p_a are considered as fixed, then an automatic adjustment of the contribution rate gives:
$$\tau = \rho \cdot d. \tag{4}$$
- If d_a and τ_a are considered as fixed and $p_a = I \cdot I_a \cdot p_a^-$ with I the general pension index and I_a an index per age, then an automatic adjustment of the pension index gives:
$$I = \tau \cdot \bar{w} \cdot \frac{\sum_a e_a \cdot N_a}{\sum_a p_a^- \cdot \lambda_a \cdot I_a \cdot N_a}. \tag{5}$$
U.S. Social Security adopts this automatic balancing if $F = 0$.
- If p_a and τ_a are considered as fixed and $\lambda_a = \left(1 - \gamma \cdot \frac{a_{\max}}{a}\right) \cdot \lambda_a^-$ where γ is a coefficient of retirement delay, then an automatic adjustment by postponing retirement ages gives:
$$\begin{aligned} \tau \cdot \sum_a w_a \cdot e_a \cdot N_a &= \sum_a p_a \cdot \left(1 - \gamma \cdot \frac{a_{\mathsf{max}}}{a}\right) \cdot \lambda_a^- \cdot N_a \\ &\Leftrightarrow \\ \alpha \cdot \sum_a p_a \cdot \frac{a_{\mathsf{max}}}{a} \cdot \lambda_a^- \cdot N_a &= \sum_a p_a \cdot \lambda_a^- \cdot N_a - \tau \cdot \sum_a w_a \cdot e_a \cdot N_a \qquad (6) \\ &\Leftrightarrow \\ \gamma &= \frac{\sum_a p_a \cdot \lambda_a^- \cdot N_a - \tau \cdot \sum_a w_a \cdot e_a \cdot N_a}{\sum_a p_a \cdot \frac{a_{\max}}{a} \cdot \lambda_a^- \cdot N_a} \end{aligned}$$
If $e_a = (1 - u_a) \cdot (1 - \lambda_a - i_a)$, then
$$\begin{aligned} \tau \cdot \sum_a w_a \cdot (1 - u_a) \cdot (1 - \lambda_a - i_a) \cdot N_a &= \sum_a p_a \cdot \lambda_a \cdot N_a \\ &\Leftrightarrow \\ \tau \cdot \sum_a w_a \cdot (1 - u_a) \cdot (1 - i_a) \cdot N_a &= \sum_a \lambda_a \cdot (p_a - \tau \cdot w_a \cdot (1 - u_a)) \cdot N_a \\ &\Leftrightarrow \end{aligned}$$

$$\gamma = \frac{\sum_a \lambda_a^- \cdot p_a - \tau \cdot w_a \cdot (1-u_a) \cdot \left(1 - i_a - \lambda_a^-\right) \cdot N_a}{\sum_a \frac{a_{\max}}{a} \cdot \lambda_a^- \cdot (p_a - \tau \cdot w_a \cdot (1-u_a)) \cdot N_a} \tag{7}$$

$f \neq 0$: the alternatives to automatic adjustmen are more numerous and it can be interesting to plan the budget balance on a large number of periods. But the thorniest question is how to plan a sequence of changes in pension parameters? That requires to identify an objective function.

Japan makes predictions every 5 years on a 95-year horizon and computes the intertemporal solvency with respect to this horizon.

From a prospective point of view, the U.S. Social Security opted for a 75-year time horizon, and the forecasts are published annually. The 75-year annual forecast of U.S. Social Security permits a thorough analysis of the solvency. Notably, they give an estimation of the year when the system reaches bankruptcy. After this critical year, in the absence of corrective measures, the federal government has an obligation to reduce pensions to achieve a financial balance between pension payments and social contributions. The adjustment is automatic and brutal because the U.S. Social Security does not have the right to borrow. The case of U.S. Social Security budgetary rule is interesting since it must comply with a rule that prohibits debt. Therefore, this means that the system can make it if the deficit has previously achieved surpluses. If $f = 0$, the adjustment is brutal because the pension scheme is in bankruptcy and can only pay pensions at the height of its revenue, which , *de facto*, means a sharp decline in pensions.

Sweden is only interested in working generations alive today and current acquired rights and the contributions they will perform in the future. The implicit prediction horizon is the maximum life length of the younger generation of workers. Sweden has adopted a full adjustment mechanism where a global index on pension benefits is used to guarantee each year an intertemporal budget balance which is estimated as an equality between the discounted sum of current and futur payroll taxes and the implicit liabilities net of the reserve fund. The notional accounts give each individual a virtual accumulated capital which is made of the sum of his contributions "virtually" revalued annually by the real growth rate of the average wage in the economy. Note that the virtual capital is discounted at i. In fact, i is a forecast of the future average growth rate of the average wage in the economy. This approximation has been set at 1.6% per annum. What mechanisms to balance the pension system? Respecting the relative standard of living of retirees is ensured by the indexation of pension capital and the growth rate of average wages. This implies two things:

- On the one hand, the forecast growth rate (discount rate equation giving the board) is accurate. From this point of view, a balancing mechanism is provided which is effective only if the reality deviates from the performance by 1.6% and pensions are adjusted accordingly[3].

- Secondly, the employment rate is stable. It is clear that any economic crisis will involve a balancing mechanism consistent (all things being equal) with the equation. This will necessarily lead to give up the indexation of pensions on the average wage increase.

This de-indexation is specifically activated when the amount of resources in the system consists in the total assets of the reserve fund and virtual assets is less than the sum of accumulated pension rights. Sunden 2009 shows the evolution of this balance ratio from 2002 to 2008. This unfavorable change led to the downward indexation of pensions. The principle of this de-indexation is to activate the reserve fund. The interesting point is probably wondering what can induce an intertemporal unbalance. In fact, at the steady state, the collected contributions should be equal to the sum of the promises made by the pension plan.

When adding up the amount of reserves, the amount of resources should exceeds liabilities by far. Therefore, it is necessary to have both a depressed economy and financial rates of return to achieve a low indexation.

In addition, what about the promised actuarial neutrality if one can change the pensions including through the discount rate (Guérin and Legros, 2002).

The challenge of the increase in life expectancy is clearly taken into account by inserting an explicit conversion rate in the actuarial formula for calculating pensions. They are actuarially neutral, varying by cohort and age at which the individual retires.

However, since the indexing rate of pensions may vary, several combinations exist between index and life expectancy which give the same conversion rate. As we have said very clearly this questions the strict actuarial neutrality regime.

The challenge of the size of generations is taken into account by the ad-

---

[3] In passing, in the most ancient texts, the case of a negative difference was not considered Sunden (1998).

justment mechanism as well as the variability of the employment rate. Lassila and Tarmo (2007) show that fertility breaks may be taken into account by the mechanism.

The Swedish model has a major limitation. When the economic and financial crisis inferred a capital loss in the reserve fund and a reduction in the growth of revenue, the Swedish capital ratio fell below the critical value of 1. The amount of pensions is reduced in order to return to balance. But the government has proposed to offset this by reducing taxes levied on pensions. In addition, the pension plan has decided to spread the adjustment over three years. We can therefore conclude that, in case of difficulty, the adjustment rules are modified in a discretionary manner. Another interpretation is that the pension system is still in control and that the crisis has raised difficulties which had not been anticipated before.

We develop a "simple" method. The objective function is defined as a quadratic loss function. Quadratic cost functions are commonly used in the analysis of monetary policy (Svensson, 2003). A similar approach applied to retirement has been developed by Berger and Lavigne (2007). But their adjustment relates solely to the contribution rate, and the social cost is measured by the square of the change in each period. Moreover, they do not introduce intertemporal discount, which leaves no possibility of procrastination. Here the approach is enhanced because it leaves two possible adjustment modes: by costs and/or by revenues. With ABM, the adjustment should result in incremental changes. Indeed, it is hoped that the AAM lead to sufficient adjustments and they contribute to a better financial balance. The ABM is an ultimate setting that can be expected to be marginal. Of course, it would be naive to think that minimizing a quadratic loss function could be sufficient to capture all the problems related to the adjustment of the pension system. However, the quadratic loss synthetizes well the will to smooth the changes in the current legislation.

The value of the loss associated to each period is measured by:

$$LF_t = \alpha \cdot (A_t - 1)^2 + (1 - \alpha) \cdot (B_t - 1)^2 . \tag{8}$$

where A_t and B_t are two deformation coefficients which modify respectively the present and future payroll taxes and pension expenditures relatively to those established by the current law. This loss function captures the fact that changing parameters is costly (both socially and politically) and that minimizing it means the social planner hopes to limit changes. The social planner sets a time horizon T to balance the sum of discounted receipts and the sum of discounted expenditures:

$$\sum_{t=1}^{T} \frac{A_t \cdot \tau_t \cdot w_t \cdot e_t \cdot L_t}{\Pi_{i=1}^{t} R_i} + F_0 = \sum_{t=1}^{T} \frac{B_t \cdot \rho_t \cdot w_t \cdot d_t \cdot L_t}{\Pi_{i=1}^{t} R_i}. \tag{9}$$

The optimizing program is:

$$\left\{ \begin{array}{l} \min\limits_{\{A_t, B_t\}} \sum_{t=1}^{T} \beta^{t-1} \cdot LF_t \\ s.t.\ \sum_{t=1}^{T} \frac{A_t \cdot \tau_t \cdot w_t \cdot e_t \cdot L_t}{{}^{t}_{i=2} R_i} + R_1 \cdot F_0 = \sum_{t=1}^{T} \frac{B_t \cdot \rho_t \cdot w_t \cdot d_t \cdot L_t}{{}^{t}_{i=2} R_i} \end{array} \right. . \tag{10}$$

The F.O.C gives:

$$\left\{ \begin{array}{l} A_t : \beta^{t-1} \cdot 2 \cdot \alpha \cdot (A_t - 1) = \psi \cdot \frac{\tau_t \cdot w_t \cdot e_t \cdot L_t}{{}^{t}_{i=2} R_i} \\ B_t : \beta^{t-1} \cdot 2 \cdot (1-\alpha) \cdot (B_t - 1) = -\psi \cdot \frac{\rho_t \cdot w_t \cdot d_t \cdot L_t}{{}^{t}_{i=2} R_i} \end{array} \right. . \tag{11}$$

where ψ is the Lagrange multiplier. We deduce that:

$$\left\{ \begin{array}{l} (A_t - 1) = \frac{\tau_t \cdot w_t \cdot e_t \cdot L_t}{\tau_1 \cdot w_1 \cdot e_1 \cdot L_1} \cdot \frac{1}{\beta^{t-1} \cdot\ {}^{t}_{i=2} R_i} \cdot (A_1 - 1) \\ (B_t - 1) = -\frac{\rho_t \cdot w_t \cdot d_t \cdot L_t}{\tau_1 \cdot w_1 \cdot e_1 \cdot L_1} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{\beta^{t-1} \cdot\ {}^{t}_{i=2} R_i} \cdot (A_1 - 1) \end{array} \right. \tag{12}$$

By incorporating these two expressions in the intertemporal budget constraint, we obtain:

$$\begin{array}{l} \sum_{t=1}^{T} \frac{\tau_t \cdot w_t \cdot e_t \cdot L_t \cdot \left(\frac{\tau_t \cdot w_t \cdot e_t \cdot L_t}{\tau_1 \cdot w_1 \cdot e_1 \cdot L_1} \frac{1}{\beta^{t-1} \cdot\ {}^{t}_{i=2} R_i} \cdot (A_1 - 1) + 1 \right)}{\Pi_{i=2}^{t} R_i} + F_0 \\ = \\ \sum_{t=1}^{T} \frac{\rho_t \cdot w_t \cdot d_t \cdot L_t \cdot \left(-\frac{\rho_t \cdot w_t \cdot d_t \cdot L_t}{\tau_1 \cdot w_1 \cdot e_1 \cdot L_1} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{\beta^{t-1} \cdot\ {}^{t}_{i=2} R_i} \cdot (A_1 - 1) + 1 \right)}{\Pi_{i=2}^{t} R_i} \end{array} \tag{13}$$

We identify A_1 and B_1 :

$$\left\{ \begin{array}{l} A_1 = 1 - \frac{\frac{F_T}{\Pi_{i=1}^{T} R_i}}{\sum_{t=1}^{T} \frac{1}{\beta^{t-1} \cdot \left(\Pi_{i=1}^{t} R_i \right)^2} \cdot \left(\frac{(\tau_t \cdot w_t \cdot e_t \cdot L_t)^2 + \frac{\alpha}{1-\alpha} \cdot (\rho_t \cdot w_t \cdot d_t \cdot L_t)^2}{\tau_1 \cdot w_1 \cdot e_1 \cdot L_1} \right)} \\ B_1 = 1 + \frac{\frac{F_T}{\Pi_{i=1}^{T} R_i}}{\sum_{t=1}^{T} \frac{1}{\beta^{t-1} \cdot \left(\Pi_{i=1}^{t} R_i \right)^2} \cdot \left(\frac{\frac{1-\alpha}{\alpha} \cdot (\tau_t \cdot w_t \cdot e_t \cdot L_t)^2 + (\rho_t \cdot w_t \cdot d_t \cdot L_t)^2}{\rho_1 \cdot w_1 \cdot d_1 \cdot L_1} \right)} \end{array} \right. \tag{14}$$

and deduce the dynamics of coefficients adjustment:

$$\begin{cases} A_t = 1 + \frac{\tau_t\cdot w_t\cdot e_t\cdot L_t}{\tau_1\cdot w_1\cdot e_1\cdot L_1}\cdot\frac{1}{\beta^{t-1}\cdot\ {}_{i=2}^{t}R_i}\cdot(A_1-1) \\ B_t = 1 + \frac{\rho_t\cdot w_t\cdot d_t\cdot L_t}{\rho_1\cdot w_1\cdot d_1\cdot L_1}\cdot\frac{1}{\beta^{t-1}\cdot\ {}_{i=2}^{t}R_i}\cdot(B_1-1) \end{cases} \tag{15}$$

This maximizing problem can be completed by adding constraints on the level of the reserve fund ($F_T > 0$ or $F_t \geq 0\ \forall t$ if no debt constraint) or the adjustment parameters.

These results can be interpreted in three ways:

- A_t and B_t can induce practical implications in terms of pension reforms. They define distances to a fixed target in terms of payroll taxes (receipts) and pension benefits (expenditures).

- Measuring A_t and B_t would show how the pension schemes are strongly unbalanced in the long run;

- Revealed preferences: reforms imply changes in legislation. The levels of expenditures and receipts are modified w.r.t a previous scenario without reform. Assuming A_t and B_t to be measured with accuracy would associate public decisions with an implicit function of public preferences.

Consider a stationary economy: $\frac{\tau_t\cdot w_t\cdot e_t\cdot L_t}{\tau_1\cdot w_1\cdot e_1\cdot L_1} = \Gamma^t = \frac{\rho_t\cdot w_t\cdot d_t\cdot L_t}{\rho_1\cdot w_1\cdot d_1\cdot L_1}\ R_t = R = \frac{1}{\beta}$. We obtain the following deformation coefficents:

$$\begin{cases} A_1 = 1 + \dfrac{\frac{1-\left(\frac{\Gamma}{R}\right)^T}{1-\left(\frac{\Gamma}{R}\right)}\cdot\ {}_{1}-R_1\cdot F_0}{(\tau_1\cdot w_1\cdot e_1\cdot L_1)} \Big/ \dfrac{\left(1-\frac{1}{\beta^T}\cdot\left(\frac{\Gamma}{R}\right)^{2T}\right)\cdot\left(1+\frac{\alpha}{1-\alpha}\cdot h_1^2\right)}{1-\frac{1}{\beta}\cdot\left(\frac{\Gamma}{R}\right)^2} \\ B_1 = 1 - \dfrac{\frac{1-\left(\frac{\Gamma}{R}\right)^T}{1-\left(\frac{\Gamma}{R}\right)}\cdot\ {}_{1}-R_1\cdot F_0}{(\tau_1\cdot w_1\cdot e_1\cdot L_1)} \Big/ \dfrac{\left(1-\frac{1}{\beta^T}\cdot\left(\frac{\Gamma}{R}\right)^{2T}\right)\cdot\left(\frac{1-\alpha}{\alpha}\cdot\frac{1}{h_1}+h_1\right)}{1-\frac{1}{\beta}\cdot\left(\frac{\Gamma}{R}\right)^2} \end{cases} \tag{16}$$

with $\Delta_1 = \rho_1\cdot w_1\cdot d_1\cdot L_1 - \tau_1\cdot w_1\cdot e_1\cdot L_1$ and $h_1 = \frac{\rho_1\cdot w_1\cdot d_1\cdot L_1}{\tau_1\cdot w_1\cdot e_1\cdot L_1}$.
We deduce then:

$$\begin{cases} A_t = 1 + \Gamma^{t-1}\cdot(A_1-1) \\ B_t = 1 + \Gamma^{t-1}\cdot(B_1-1) \end{cases}. \tag{17}$$

As mentioned earlier, the U.S. Social Security publishes annual forecasts with a 75-year horizon. This forecast comprises three scenarios: pessimistic, optimistic and middle. This publication is important because it gives a clear idea of the likely survival duration of the pension system. In this section, we look at what the use of ABM requires in terms of increased revenue and spending cuts.

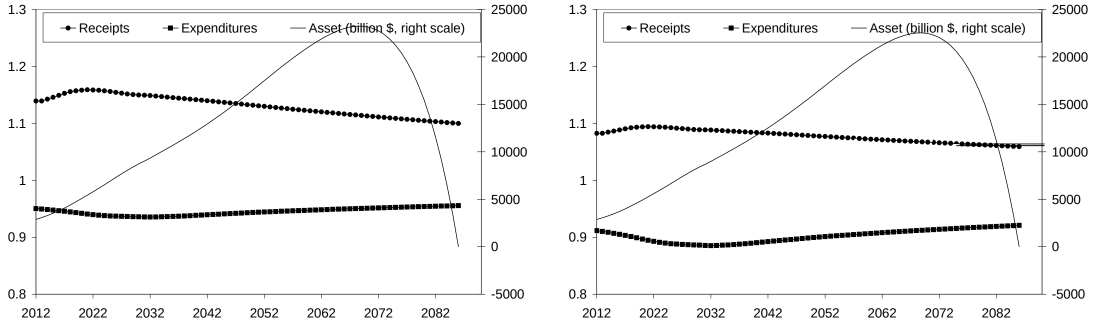
We consider several parametric variants:

- the forecast horizon;
- the time preference (or degree of procrastination);
- the weight of social adjustment by revenues (vs. expenses).

Figures 1, 2 and 3 respectively show parametric variants with forecast horizons of 75, 50 and 25 years. The U.S. pension system performs surpluses until 2032 (intermediate scenario forecasting). After this date, the U.S. government will be forced to reform (tax increase or decrease in pensions). The longer the horizon, the more the planner integrates imbalance. This means that the adjustments are very sensitive to the horizon. For a 25-year horizon, the present value of the unfunded fraction of the liabilities is low. It increases with the forecast horizon. Thus, the need for adjustments are very low (A and B near 1 at $T = 25$ (Figs 3). In contrast when $T = 50$ (Figs 2), $|A - 1|$ and $|B - 1|$ can vary between 5 and 15%. For $T = 50$ (Figs 1), $|A - 1|$ and $|B - 1|$ may vary up to 30% depending on the parametric choice.

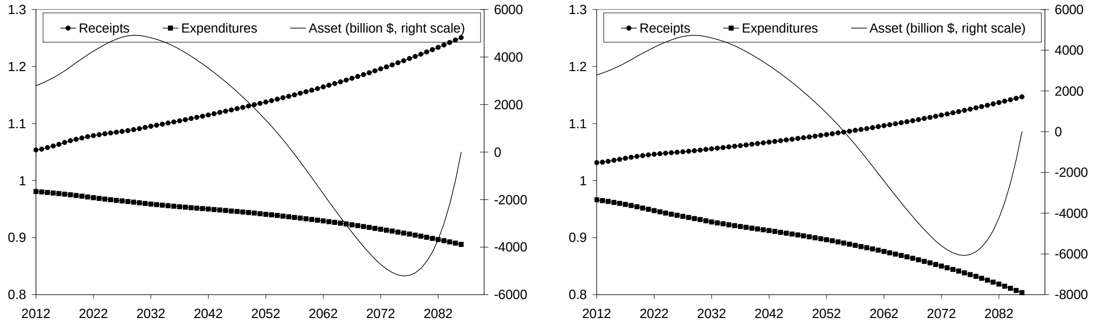
Variations of time preference (Fig x.1.y for $\beta = 1$, Fig x.2.y for $\beta = 0.975$ and Fig x.3.y for $\beta = 0.95$) clearly show the consequences of postponing adjustment mechanisms. Delaying adjusment induces very high adjustment costs in the future.

Figures Fig. x.y.1 ($\alpha = 0.25$) and Fig. xy2 ($\alpha = 0.5$) show the profile of A and B for variants of the social weighting (given respectively to revenue and expenditure). Not surprisingly, the adjustment of expenses is more demanding for high values of α.



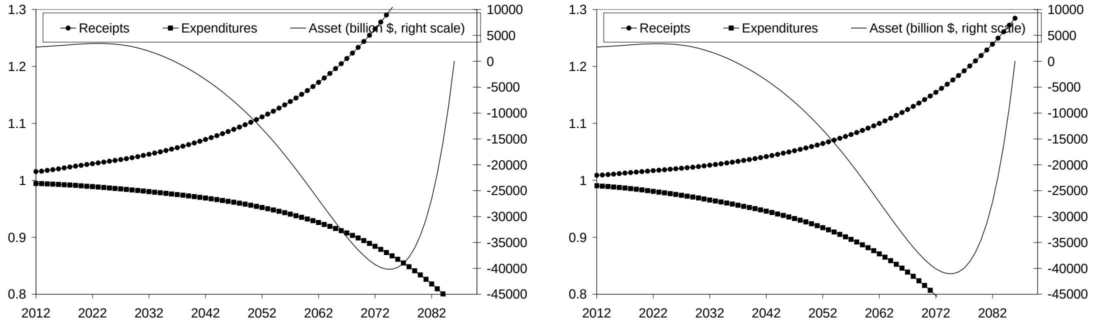
111. $\alpha = 0.25$ 112. $\alpha = 0.5$

$\beta = 1$



121. $\alpha = 0.25$ 122. $\alpha = 0.5$

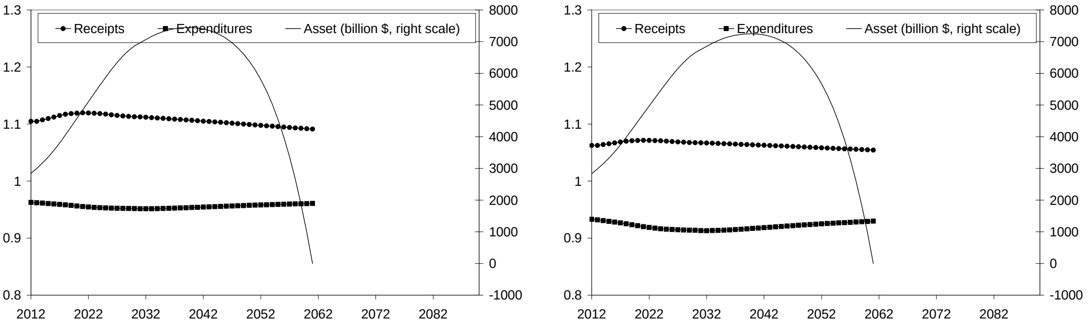
$\beta = 0.975$



131. $\alpha = 0.25$ 132. $\alpha = 0.5$

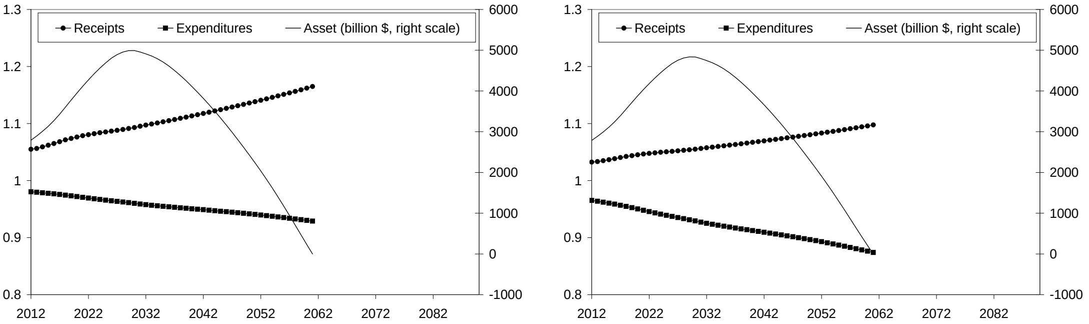
$\beta = 0.95$

Figures 1. Time horizon $T = 75$



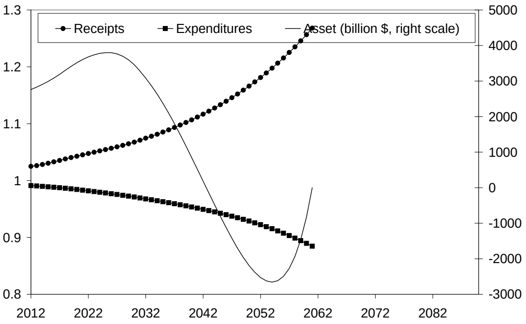
211. $\alpha = 0.25$ 212. $\alpha = 0.5$

$\beta = 1$



221. $\alpha = 0.25$ 222. $\alpha = 0.5$

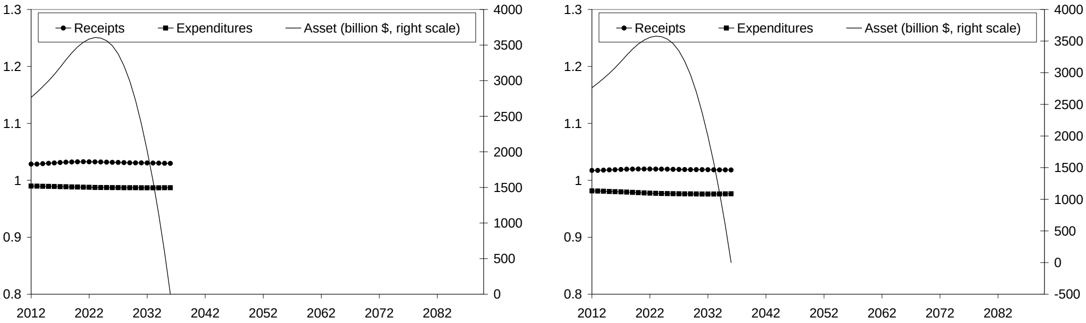
$\beta = 0.975$



231. $\alpha = 0.25$ 232. $\alpha = 0.5$

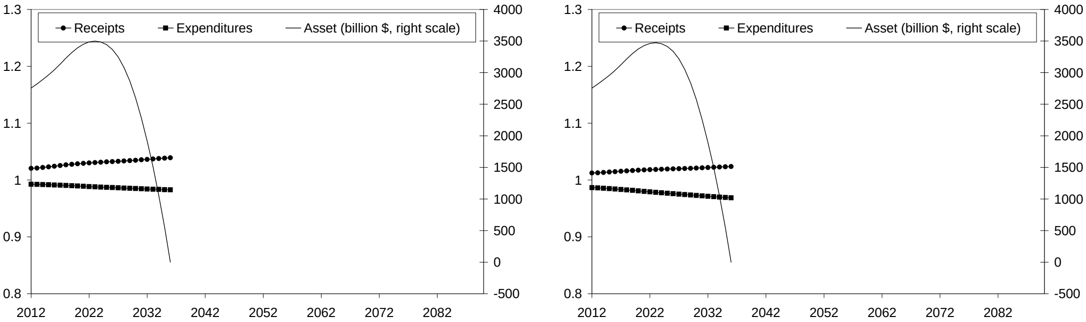
$\beta = 0.95$

Figures 2. Time horizon $T = 50$



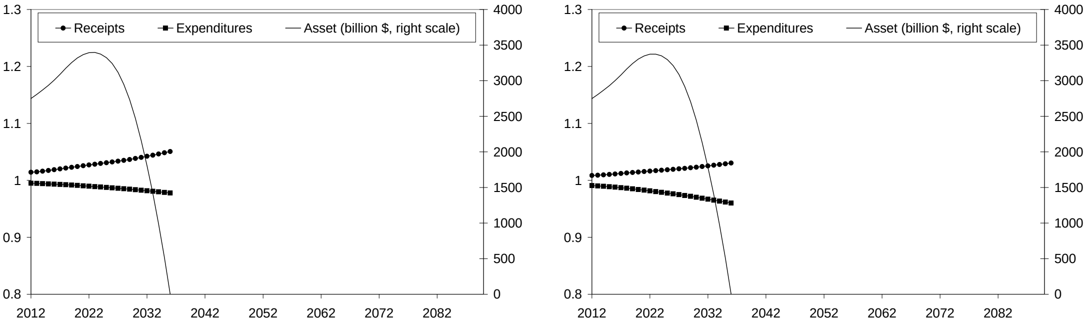
311. $\alpha = 0.25$ 312. $\alpha = 0.5$

$\beta = 1$



321. $\alpha = 0.25$ 322. $\alpha = 0.5$

$\beta = 0.975$

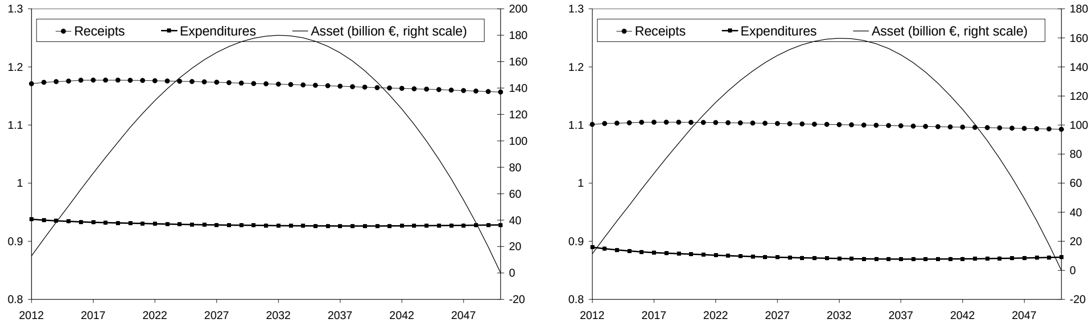


331. $\alpha = 0.25$ 332. $\alpha = 0.5$

$\beta = 0.95$

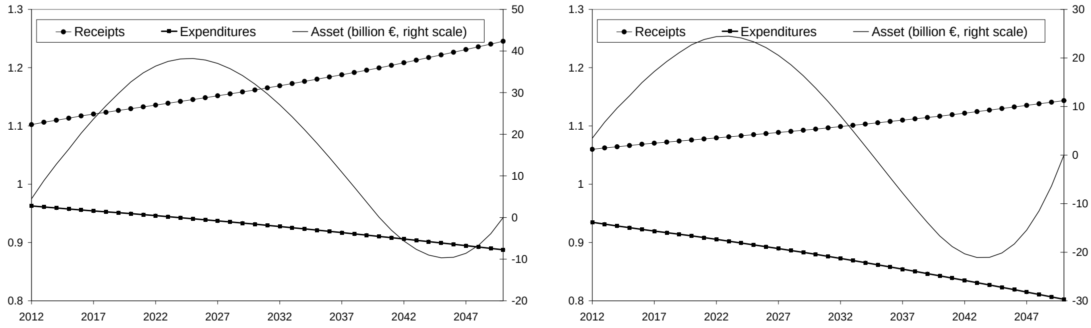
Figures 3. Time horizon $T = 25$

The French system is currently in deficit, and forecasts for 2050 are rather alarmist (COR, 8th report, 2010, scenario B). We do the same exercises to estimate an ABM. Three variants for the horizon are performed: $T = 39$, $T = 26$ and $T = 13$. Sensitivity results are similar to those obtained with the U.S. pension system.



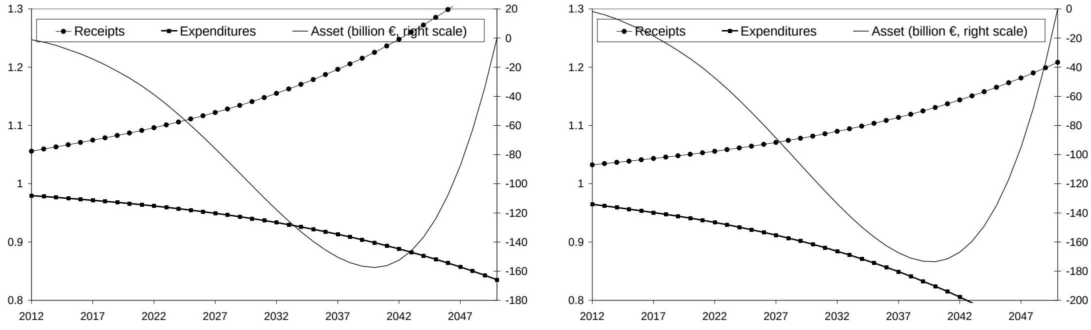
411. $\alpha = 0.25$ 412. $\alpha = 0.5$

$\beta = 1$



421. $\alpha = 0.25$ 422. $\alpha = 0.5$

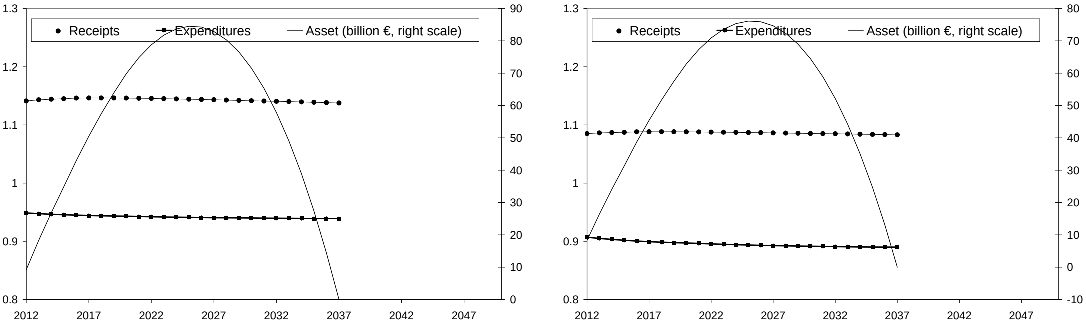
$\beta = 0.975$



431. $\alpha = 0.25$ 432. $\alpha = 0.5$

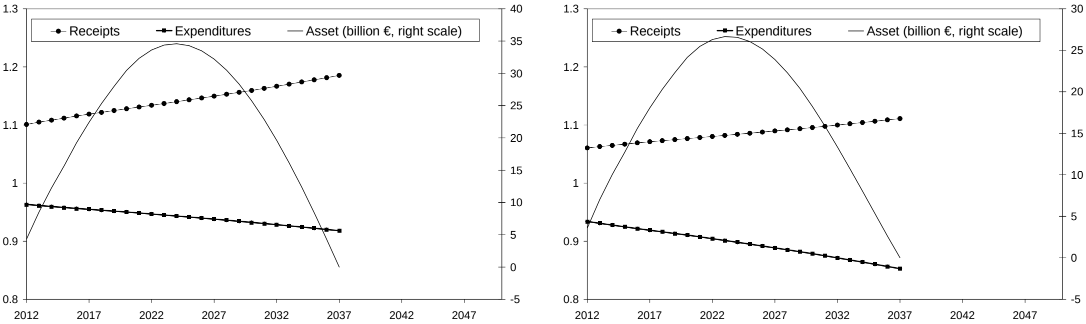
$\beta = 0.95$

Figures 4. Time horizon $T = 39$



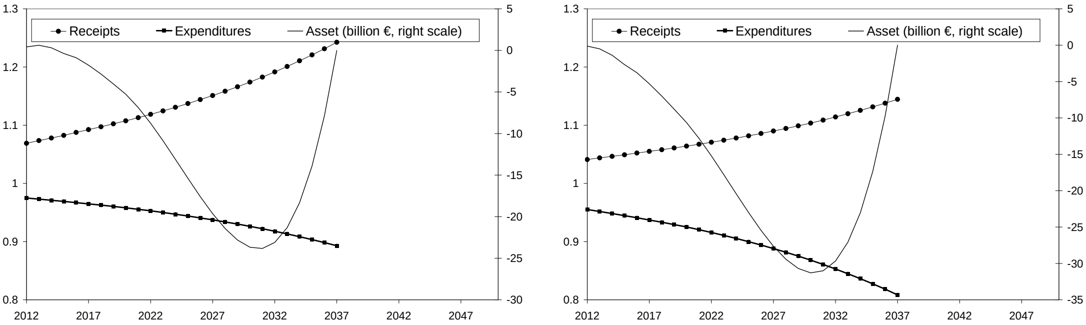
511. $\alpha = 0.25$ 512. $\alpha = 0.5$

$\beta = 1$



521. $\alpha = 0.25$ 522. $\alpha = 0.5$

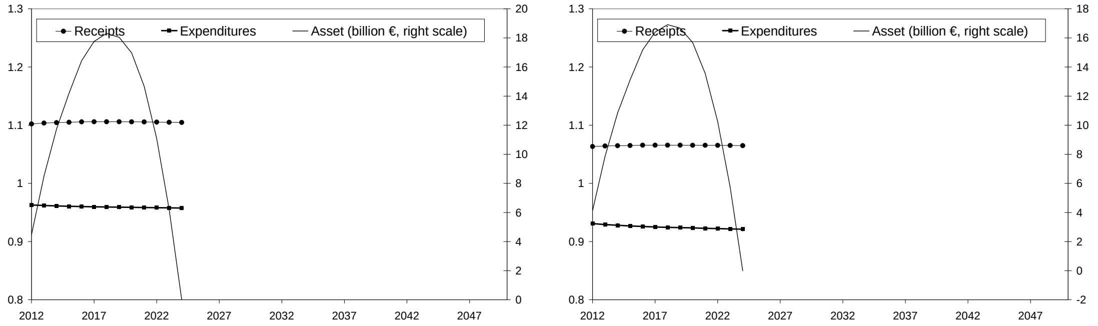
$\beta = 0.975$



531. $\alpha = 0.25$ 532. $\alpha = 0.5$

$\beta = 0.95$

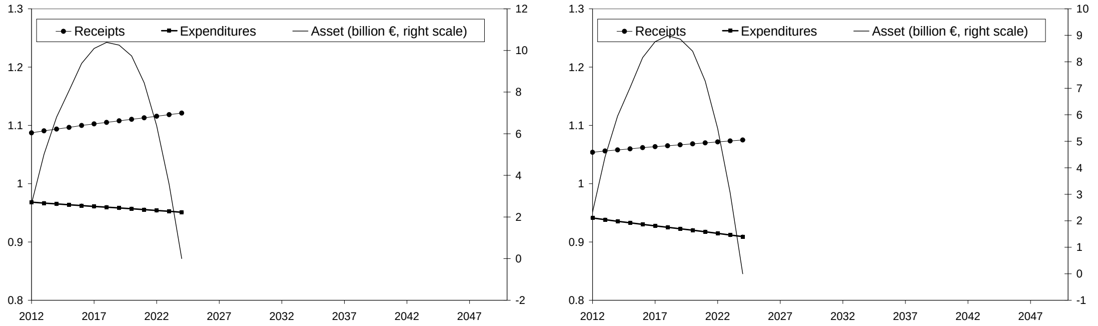
Figures 5. Time horizon $T = 26$



611. $\alpha = 0.25$

612. $\alpha = 0.5$

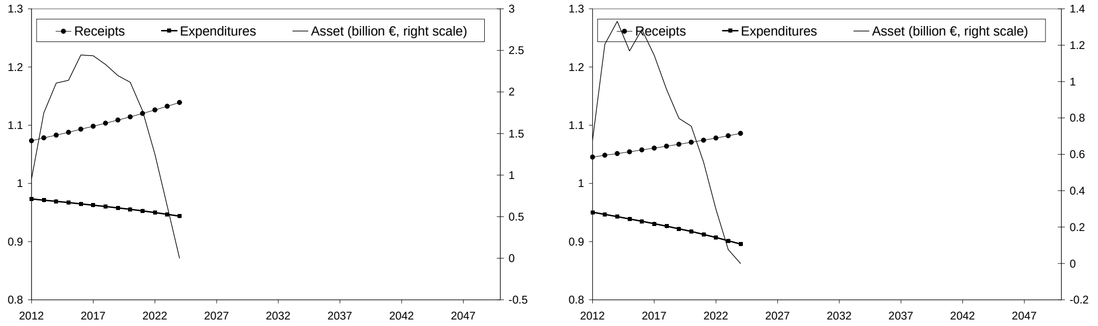
$\beta = 1$



621. $\alpha = 0.25$

622. $\alpha = 0.5$

$\beta = 0.975$



631. $\alpha = 0.25$

632. $\alpha = 0.5$

$\beta = 0.95$

Figures 6. Time horizon $T = 13$

This article has identified different types of AAM that can be implemented and has shown how they contribute to a better solvency. Sweden is the only country that strengthens its AAM with an ABM that ensures financial stability. Similarly as in the Swedish pension, we propose to build an ABM starting from a dynamic programming setting. For a given planning horizon, we obtain formulas that determine how at each period revenues and expenses must be adjusted. That allows to consider the ABM chosen by Sweden as a special case. Indeed, the Swedish ABM can be obtained by assuming very high adjustment costs on revenue and choosing a particular concept of measure of solvency. We apply these formulas to the financial balances of the American and French systems. Using dynamic programming avoids brutal adjustments and thus moderates or smooths the marginal adjustments necessary for financial stability.

However, this exercise of dynamic programming presents a major limit. In effect, the ABM is evaluated in a context where economic variables are assumed to be exogenous. A study of the relationship between the fitting parameters and the evolution of the economy can be a natural extension of this article. From a macroeconomic point of view, OLG-CGE models have been developed to estimate the impact of Social Security reforms in an intertemporal and intergenerational general equilibrium framework. Such models are used to "optimize" Social Security reforms. Furthermore, dynamic microsimulation models give a lot of details on the microeconomic impacts of Social Security reforms.

[1] Barr, Nicholas and Peter Diamond, 2011, "Improving Sweden's Automatic Pension Adjustement Mechanism", *Issue in brief*, Center for retirement research at Boston College, n°2.

[2] Berger, Charlie and Anne Lavigne, 2007, "A model of the French pension reserve fund: what could be the optimal contribution path rate?, *Journal of pensions economics and finance*, 6 (3): 233-250.

[3] Blanchet, Didier and Florence Legros, 2002, "France: The Difficult Path to Consensual Reforms", in *Social Security Pension Reform in Europe* (Martin Feldstein and Horst Siebert, Eds), University of Chicago Press.

[4] Blanchet, Didier and Jean-François Ouvrard, 2007, "Indicateurs d'engagements implicites des systèmes de retraite : que mesurent-ils exactement?", *Revue française d'économie*, 22 (1):3-47.

[5] Boado-Penas, María del Carmen, Inmaculada Dominguez-Fabian and Carlos Vidal-Meliá, 2007, "Notional Defined Contribution Accounts (NDCs): Solvency and Risk, Application to the Case of Spain", *International Social Security Review*, 60 (4), 105-127.

[6] Boado-Penas, María del Carmen, Carlos Vidal-Meliá and Junichi Sakamoto, 2010, "Models of the Actuarial Balance of the Pay-As-You-Go Pension System. A Review and some Policy Recommendations", mimeo.

[7] Bosworth Barry and Kent Weaver, 2011, "Social Security on Auto-Pilot: International Experience With Automatic Mechanisms", mimeo.

[8] Capretta, James C., 2006, "Building Automatic Solvency into U.S. Social Security: Insights from Sweden and Germany", *Policy brief*, Brooking Institutions, n°151.

[9] Ehnsson, Gudrun (Ed.), 2012, *Orange Report: Annual Report of the Swedish Pension System 2011*, Swedish Social Insurance Agency.

[10] Fujisawa, Yosuke and Johnny Siu-Hang Li, 2012, "The Impact of the Automatic Balancing Mechanism for the Public Pension in Japan on the Extreme Elderly", *North American Actuarial Journal*, 16 (2).

[11] Garcia, Manuel, Juan M. Nave and Carlos Vidal-Meliá, 2011, "The US actuarial balance model for the pay-as-you-go system and its application to Spain", mimeo.

[12] Guérin, Jean-Louis, Legros, Florence, 2002, Neutralité actuarielle, un concept élégant mais délicat à mettre en œuvre, *Revue d'économie financière*, n°68.

[13] Lassila, Jukka and Valkonen Tarmo, 2007, Mettre un frein suédois sur les retraites, in *Les retraites, libres opinions d'experts européens*, F. Legros ed., Economica.

[14] Scherman, Karl Gustaf, 2007, "The Swedish NDC system – A critical assessment", 2nd PBSS Colloquium — Helsinki, Finland — 21-23 May.

[15] Selén, Jan and Ann-Charlotte Ståhlberg, "Why Sweden's pension reform was able to be successfully implemented", *European Journal of Political Economy*, 23 (4): 1175–1184.

[16] Settergren, Ole, 2001, "The Automatic Balance Mechanism of the Swedish Pension System", working paper, National Social Insurance Board.

[17] Settergren, Ole, Mikula, Boguslaw D., 2005, "The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest", *Journal of Pension Economics and Finance*, 4 (2) : 115–138.

[18] Steuerle, C. Eugene and Rudolph Penner, 2005, "Time to Take the Federal Budget Off of Auto-Pilot," *Investor's Business Daily* (Investors.com), August 29.

[19] Sunden, Annika, 1998, "The swedish NDC pension reform", *Annals of public and cooperative economics*, vol 69, n°4.

[20] Sunden, Annika, 2009, The swedish pension system and the economic crisis, Center for retirement research at Boston College, N° 9-25, december.

[21] Svensson, Lars E.O., 2003,"The Inflation Forecast and the Loss Function." In *Central Banking, Monetary Theory and Practice: Essays in Honour of Charles Goodhart* (P. Mizen, Ed.), vol. I, Edward Elgar.

[22] Turner, John A., 2009, *Social security financing: Automatic adjustments to restore solvency*, AARP Public policy institute Research report, February.

[23] Vidal-Melia, Carlos and María del Carmen Boado-Penas, 2010 "Notes on Using the Hidden Asset or Contribution Asset to Compile the Actuarial Balance for Pay-As-You-Go Pension Systems", mimeo, 27 pages.

[24] Vidal-Melia, Carlos, María del Carmen Boado-Penas and Ole Settergren, 2009 "Automatic Balance Mechanisms in Pay-As-You-Go Pension Systems", *Geneva Papers on Risk and Insurance - Issues and Practice*, 34(2):287-317.